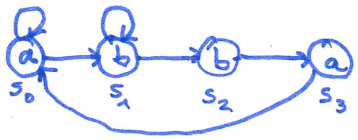
Danny MILOSAVLJEVIC
Mat. 0826039

6. Ü. Systemverifikation
Model Checking

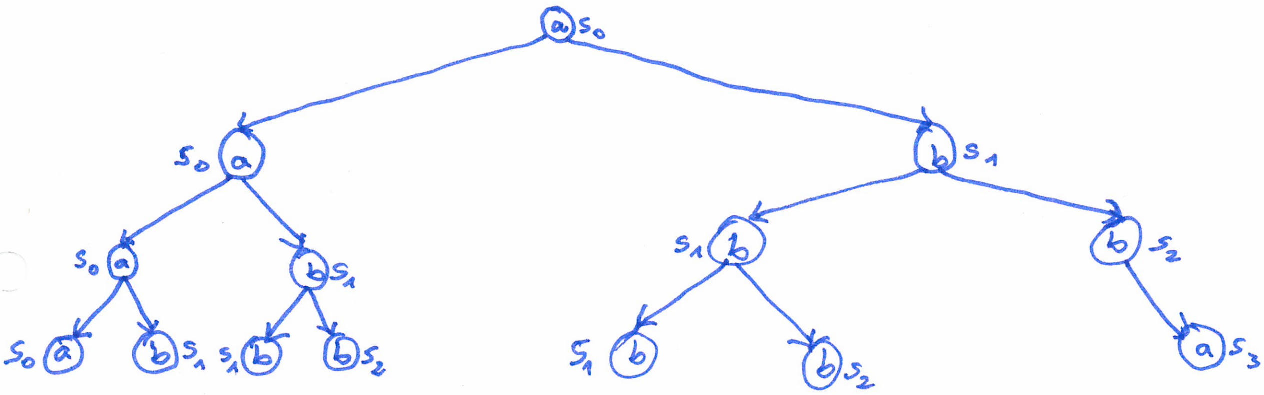
07.06.2015

Consider



1. Fix s_0 as the initial state and give the computation tree for three steps.

I assume a „step" is following an arrow.



2. Describe formulas in natural language. For each formula, in which states of the Kripke structure does it hold? (no special initial state)

(a) $a \wedge X a$ (path formula)

„a holds now and one step later".

From s_0: holds when following the cycle, not otherwise.

From s_3: holds.

(b) $A G (b \, U \, a)$ „All paths will globally in the future reach a state where a holds after passing 0 or more states where b held before"

(π) $\neg(\text{true} \; U \; \neg(b \; U \; a))$

$\neg(\exists k \geq 0 \wedge \pi^k \models \neg(b \; U \; a)))$

$\forall k \, (k < 0 \vee \pi^k \not\models \neg(b \; U \; a))$

$\forall k \, (k < 0 \vee \underbrace{\pi^k \models b \; U \; a}_{\exists k' (k' \geq 0 \wedge (\pi^k)^{k'} \models a \wedge \forall j (0 \leq j < k' \Rightarrow (\pi^k)^j \models b))})$

Since each path eventually contains $b^* a$, it will eventually be reached by π^k.
So the formula always holds regardless of initial state.

Danny MILOSAVLJEVIC — 6. Ü. Systemverifikation — 07.06.2015
Mat. 0826035 — Model Checking

2. (c) $E(G\ a)$ „There is a path (starting from a state s) where a holds globally in the future"

$E(\neg F \neg a)$

$E(\neg(true\ U\ \neg a))$

$E(\neg \exists k (k \geq 0 \wedge \pi^k \models \neg a))$

$E(\forall k\ \neg(k \geq 0 \wedge \pi^k \models \neg a))$

$E(\forall k (k < 0 \vee \pi^k \not\models \neg a))$

$E(\forall k (k < 0 \vee \pi^k \models a))$

Holds from s_0 and holds from s_3 (in a path thats cycles from s_0 to s_0 again and again).

2. (d) $A(G((F\neg a) \rightarrow Fb))$

$\neg(true\ U\ \neg((F\neg a) \rightarrow Fb))$

$\neg(\exists k (k \geq 0 \wedge \pi^k \models \neg((F\neg a) \rightarrow Fb)))$

$\forall k (k < 0 \vee \pi^k \not\models \neg((F\neg a) \rightarrow Fb)))$

$\forall k (k < 0 \vee \pi^k \models ((F\neg a) \rightarrow Fb))$

Case analysis: $F\neg a$ ~~will eventually be true~~.

Case 1: $F\neg a$

Then ~~it~~ Fb since the path ~~contains all~~ traverses all states, including ones where b. So $(F\neg a) \rightarrow Fb$.

Case 2: $\neg F\neg a$

Then $(F\neg a) \rightarrow Fb$.

Hence $(F\neg a) \rightarrow Fb$.

So ~~it~~ (d) holds regardless of initial state.

Danny MILOSAVLJEVIC Ü. Systemverifikation 7.06.2015
Mat. 0826039 Model Checking

3. (a) The ~~sun~~ sun „rises" infinitely often.

$AG(AF\ rises)$

Why not $AGF\ rises$? Doesn't A already traverse all the paths? What if it rises infinitely often in just one path?

(b) It is always the case that if the „sun" is shining then it's not dark.

$AG(sun \rightarrow \neg dark)$

(c) One does not make the same „mistake" twice in life.

$\neg EF(mistake \wedge EX\ mistake)$

$AG\neg(mistake \wedge EX\ mistake)$

(the X is in order to prevent F from picking up the first mistake)

(d) Austria „wins" the song contest five times in a row, somewhere in the future?

~~$EF(wins \wedge X\ wins \wedge XX\ wins \wedge XXX\ wins \wedge XXXX\ wins)$~~

$EF(wins \wedge EX(wins \wedge EX(wins \wedge EX(wins \wedge EX(wins)))))$

(e) Whenever Austria „wins" the song contest, it will be „hosted" in Austria the year after.

~~AG(wins~~

$AG(wins \rightarrow AX\ hosted)$