
Preface

The ideas and concepts of physics are best expressed in the language of mathematics. But this language is far from unique. Many different algebraic systems exist and are in use today, all with their own advantages and disadvantages. In this book we describe what we believe to be the most powerful available mathematical system developed to date. This is *geometric algebra*, which is presented as a new mathematical tool to add to your existing set as either a theoretician or experimentalist. Our aim is to introduce the new techniques via their applications, rather than as purely formal mathematics. These applications are diverse, and throughout we emphasise the unity of the mathematics underpinning each of these topics.

The history of geometric algebra is one of the more unusual tales in the development of mathematical physics. William Kingdon Clifford introduced his geometric algebra in the 1870s, building on the earlier work of Hamilton and Grassmann. It is clear from his writing that Clifford intended his algebra to describe the geometric properties of vectors, planes and higher-dimensional objects. But most physicists first encounter the algebra in the guise of the Pauli and Dirac matrix algebras of quantum theory. Few then contemplate using these unwieldy matrices for practical geometric computing. Indeed, some physicists come away from a study of Dirac theory with the view that Clifford's algebra is inherently quantum-mechanical. In this book we aim to dispel this belief by giving a straightforward introduction to this new and fundamentally different approach to vectors and vector multiplication. In this language much of the standard subject matter taught to physicists can be formulated in an elegant and highly condensed fashion. And the portability of the techniques we discuss enables us to reach a range of advanced topics with little extra work.

This book is intended to be of interest to both students and researchers in physics. The early chapters grew out of an undergraduate lecture course that we have run for a number of years in the Physics Department at Cambridge Uni-

versity. We are indebted to the students who attended the early versions of this course, and helped to shape the material into a form suitable for undergraduate tuition. These early chapters require little more than a basic knowledge of linear algebra and vector geometry, and some exposure to classical mechanics. More advanced physical concepts are introduced as the book progresses.

A number of themes run throughout this book. The first is that geometric algebra enables us to express fundamental physics in a language that is free from coordinates or indices. Coordinates are only introduced later, when the geometry of a given problem is clear. This approach gives many equations a degree of clarity which is lost in tensor algebra. A second theme is the way in which rotations are handled in geometric algebra through the use of *rotors*. This approach extends to arbitrary spaces the idea of using a complex phase to rotate in a plane. Rotor techniques can be applied in spaces of arbitrary signature and are particularly well suited to formulating Lorentz and conformal transformations. The latter are central to our treatment of non-Euclidean geometry. Rotors also provide a framework for studying Lie groups and Lie algebras, and are essential to our discussion of gauge theories.

The third theme is the invertibility of the geometric product of vectors, which makes it possible to divide by a vector. This idea extends to the vector derivative, which has an inverse in the form a first-order Green's function. The vector derivative and its inverse enable us to extend complex analytic function theory to arbitrary dimensions. This theory is perfectly suited to electromagnetism, as all four Maxwell equations can be combined into a single spacetime equation involving the invertible vector derivative. The same vector derivative appears in the Dirac theory, and is central to the gauge treatment of gravitation which dominates the final two chapters of this book.

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not complaining about the lost evenings as I worked on this book. I promise to finish the decorating now it is complete.

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