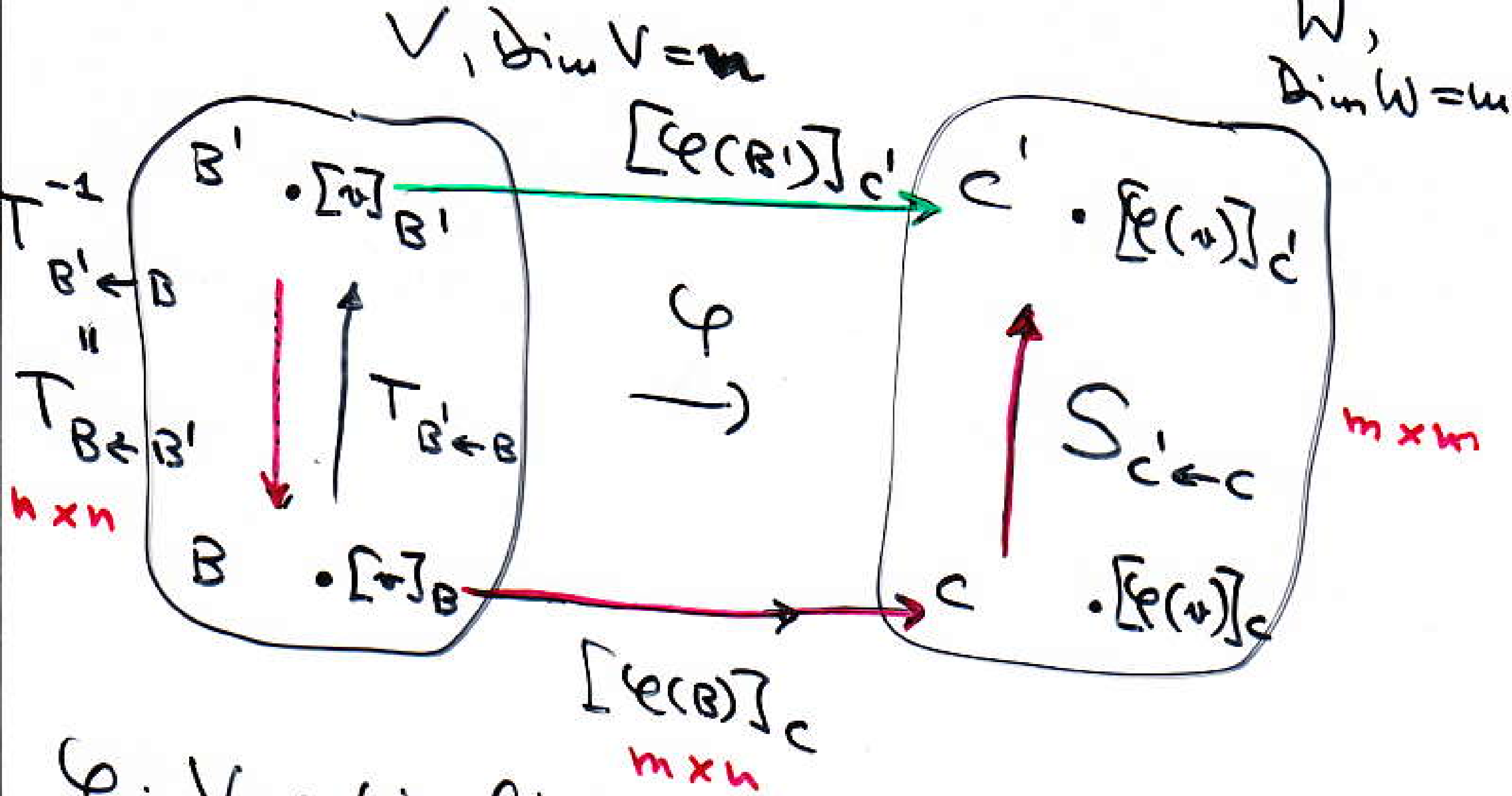

$\varphi : V \to W$ linear

Es gilt $[\varphi(v)]_C = [\varphi(B)]_C [v]_B$

Wie sieht die Matrix $[\varphi(B')]_{C'}$ aus,

$$[\varphi(v)]_{C'} = [\varphi(B')]_{C'} [v]_{B'} \Leftrightarrow$$

$$\underbrace{\boxed{[\varphi(B')]_{C'}}}_{m \times n} \cdot [v]_{B'} = S_{C' \leftarrow C} [\varphi(v)]_C =$$

$$= S_{C' \leftarrow C} [\varphi(B)]_C [v]_B =$$

$$= \boxed{S_{C' \leftarrow C} [\varphi(B)]_C T^{-1}_{B' \leftarrow B}} [v]_{B'}$$

$$\underbrace{m \times \cancel{n}\ \cancel{n} \times \cancel{n}\ \cancel{n} \times n}_{m \times n}$$

Beispiel $V = \mathbb{R}^2$

$$B = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right\} = \{ b_1, b_2 \}$$

$$B' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} = \{ b_1', b_2' \} = E_2$$

Stelle $T_{B' \leftarrow B}$ auf.

$$\left\{ \begin{aligned} \begin{pmatrix} 1 \\ 2 \end{pmatrix} &= t_{11} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t_{21} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} t_{11} \\ t_{21} \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = [b_1]_{E_2} \\ \begin{pmatrix} -1 \\ 3 \end{pmatrix} &= t_{12} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t_{22} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \end{aligned} \right.$$

$$\Downarrow$$

$$\begin{pmatrix} t_{12} \\ t_{22} \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} = [b_2]_{E_2}$$

$$T_{B' \leftarrow B} = T_{E_2 \leftarrow B} = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

Def. 3.5

V, $\dim V = n$ $\qquad$ V, $\dim V = n$

$$\begin{tikzcd}
\bullet B' \arrow[d, shift right, "T^{-1}_{B' \leftarrow B}"'] \arrow[rr, "{[\varphi(B')]_{B'}}"] & & \bullet B' \\
 & \xrightarrow{\varphi} & \\
\bullet B \arrow[u, shift right, "T_{B' \leftarrow B}"'] \arrow[rr, "{[\varphi(B)]_B}", "n \times n"'] & & \bullet B \arrow[uu, "T_{B' \leftarrow B}"']
\end{tikzcd}$$

$n \times n$ $\qquad$ $n \times n$

$$\boxed{\underset{n\times n}{[\varphi(B')]_{B'}} = T_{B'\leftarrow B}\,\underset{n\times n}{[\varphi(B)]_{B}}\,T^{-1}_{B'\leftarrow B}}$$

Kapitel 4:

Determinanten

Sei $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in \mathbb{R}^{2\times 2} \Rightarrow$

$$\boxed{\det(A) := a_{11}a_{22} - a_{12}a_{21}}$$

$$A_1 = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \Rightarrow \det(A_1) = 1\cdot 3 - (-2\cdot 4) = 3 + 8 = \underline{\underline{11}}$$

Sei $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \in \mathbb{R}^{3\times 3} \Rightarrow$

$$\boxed{\begin{aligned}\det(A) := {}& a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} \\ & - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{21}a_{12}a_{33})\end{aligned}}$$

Klaxer:

$$\left(\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right) \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix}$$

$$\mathrm{Det}(A) = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} +$$

$$+ a_{13}a_{21}a_{32} -$$

$$- (a_{13}a_{22}a_{31} + a_{32}a_{23}a_{11} +$$

$$a_{33}a_{21}a_{12})$$

<u>Beispiel</u>

- $A_2 = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 0 & -1 \\ 3 & 2 & 4 \end{pmatrix} \Rightarrow$

$$\mathrm{Det}(A_2) = \cancel{1 \cdot 0 \cdot 4} + (-2)\,2\,1 + \cancel{0 \cdot (-1) \cdot 3}$$

$$- (\cancel{3 \cdot 0 \cdot 1} + 2(-1)1 + \cancel{(-2) \cdot 0 \cdot 4})$$

$$= -4 + 2 = -2$$

$$\cdot\ A_3 = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 0 & -2 \\ 3 & 2 & 5 \end{pmatrix} \Rightarrow$$

$$\mathrm{Det}(A_3) = \cancel{1\cdot 0 \cdot 5} + (-2)\cdot 2 \cdot 1 + \cancel{0(-2)3}$$

$$- \left(\cancel{3\cdot 0\cdot 1} + 2\cdot(-2)\cdot 1 + (-2)\cancel{\cdot 0 \cdot 5}\right)$$

$$= -4 + 4 = \underline{\underline{0}}$$

<u>Beispiel 4.5</u>

$$A = \begin{pmatrix} \frac{3}{5} & \frac{2}{7} & -\frac{4}{5} \\ 1 & 0 & -2 \\ -2 & \frac{3}{7} & 3 \end{pmatrix}$$

$$\mathrm{Det}(A) \overset{5z_1}{=} \frac{1}{5}\,\mathrm{Det}\begin{pmatrix} 3 & \frac{10}{7} & -4 \\ 1 & 0 & -2 \\ -2 & \frac{3}{7} & 3 \end{pmatrix} =$$

$$\overset{7s_2}{=} \frac{1}{5}\cdot\frac{1}{7}\,\mathrm{Det}\begin{pmatrix} 3 & 10 & -4 \\ 1 & 0 & -2 \\ -2 & 3 & 3 \end{pmatrix} =$$

$3\ 0\ -6$

$2\ 0\ -4$

$$\overset{z_1 \leftrightarrow z_2}{=} (-1)\,\frac{1}{5}\,\frac{1}{7}\,\mathrm{Det}\begin{pmatrix} 1 & 0 & -2 \\ 3 & 10 & -4 \\ -2 & 3 & 3 \end{pmatrix} =$$

$$\overset{\substack{z_2 - 3z_1 \\ z_3 + 2z_1}}{=} (-1)\,\frac{1}{5}\,\frac{1}{7}\,\mathrm{Det}\begin{pmatrix} 1 & 0 & -2 \\ 0 & 10 & 2 \\ 0 & 3 & -1 \end{pmatrix} =$$

$S_2 \leftrightarrow S_3$

$$= (-1)(-1)\,\frac{1}{5}\cdot\frac{1}{7}\,\mathrm{Det}\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 10 \\ 0 & -1 & 3 \end{pmatrix} =$$

$z_2 \leftrightarrow z_3$

$$= (-1)(-1)(-1)\,\frac{1}{5}\cdot\frac{1}{7}\,\mathrm{Det}\begin{pmatrix} 1 & -2 & 0 \\ \overset{0}{0} & \overset{-2}{-1} & \overset{6}{3} \\ 0 & 2 & 10 \end{pmatrix} =$$

$z_3 + 2z_2$

$$= (-1)(-1)(-1)\,\frac{1}{5}\cdot\frac{1}{7}\,\underbrace{\mathrm{Det}\begin{pmatrix} 1 & -2 & 0 \\ 0 & -1 & 3 \\ 0 & 0 & 16 \end{pmatrix}}_{= 1\cdot(-1)\cdot 16} =$$

Δ-Matrix

$$= (-1)(-1)(-1)\,\frac{-16}{35} = \frac{16}{35}.$$