Beispiel 7.3.

Löse $y' - Ay = \emptyset$, $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, $t \in [0,T]$

1. Schritt $\quad p(\lambda) = \mathrm{Det}(A - \lambda I) = \emptyset$

$$\lambda_1 = 1, \; \lambda_2 = 3$$

2. Schritt $\quad v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} s, \; s \neq 0 \;\; || \;\; v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} s, \; s \neq 0$

3. Schritt

$$y(t) = \alpha_1 e^{\lambda_1 t} v_1 + \alpha_2 e^{\lambda_2 t} v_2 \iff$$

$$y(t) = Y(t)\alpha = \underbrace{\begin{pmatrix} e^{\lambda_1 t} v_1 & e^{\lambda_2 t} v_2 \\ | & | \end{pmatrix}}_{FM} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$$

mit

$$Y(t) = \left(e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} \vdots \; e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) = \begin{pmatrix} e^t & e^{3t} \\ -e^t & e^{3t} \end{pmatrix}.$$

Es gilt

$$\mathrm{Det}\, Y(t) = e^{4t} + e^{4t} = 2e^{4t} \neq 0 \;\forall t$$

$$\boxed{y(t) = \alpha_1 e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \alpha_2 e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \;, \alpha_1, \alpha_2 \in \mathbb{R}$$

noch nicht ?

AWP : DG + AB: $y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow y(t)$ wird ?

$t=0$

$$y(0)=\begin{pmatrix}1\\1\end{pmatrix}=\alpha_1\begin{pmatrix}1\\-1\end{pmatrix}+\alpha_2\begin{pmatrix}1\\1\end{pmatrix},\ \alpha_1=\emptyset,\ \alpha_2=1$$

$$\Rightarrow \boxed{y(t)=e^{3t}\begin{pmatrix}1\\1\end{pmatrix}}.$$

A nicht diagonal.

$A\in\mathbb{R}^{2\times 2}$

Es sei

$$X^{-1}AX=J=\begin{pmatrix}\lambda & 1\\ \emptyset & \lambda\end{pmatrix},$$

$\Downarrow$

$$\boxed{A=XJX^{-1}} \qquad X=\begin{pmatrix} | & | \\ x & \ell \\ | & | \end{pmatrix}.$$

HV

EV

Die Dgl lautet

$\dot y(t)-Ay(t)=\emptyset$ gekoppelt! $\Rightarrow$

$$X^{-1}\dot y-X^{-1}XJX^{-1}y=\emptyset \iff$$

$$\underbrace{X^{-1}\dot y}_{\dot v}-J\underbrace{X^{-1}y}_{v}=\emptyset \text{ mit } v:=X^{-1}y \Rightarrow$$

$$\Rightarrow \boxed{\dot v-Jv=\emptyset} \text{ teilw. entkoppelt}$$

$$\Downarrow\ \boxed{y=Xv}.$$

$$\begin{pmatrix} \dot{v}_1(t) \\ \dot{v}_2(t) \end{pmatrix} - \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = \emptyset$$

2. Gl.: $\dot{v}_2(t) - \lambda v_2(t) = \emptyset \Rightarrow \underline{\underline{v_2(t) = e^{\lambda t}\alpha_2}}$

1. Gl.: $\dot{v}_1(t) - \lambda v_1(t) - v_2(t) = \emptyset \Leftrightarrow$

(*) $\boxed{\dot{v}_1(t) = \lambda v_1(t) + e^{\lambda t}\alpha_2}$

Es gilt: ~~Die~~ Lösg $v_1(t)$ hat die Form

$\boxed{v_1(t) = e^{\lambda t}\alpha_1 + te^{\lambda t}\alpha_2}$

Einsetzen in (*):

Probe: $L = \dot{v}_1(t) = \lambda \cancel{e^{\lambda t}}\alpha_1 + \cancel{e^{\lambda t}}\alpha_2 + \lambda t \cancel{e^{\lambda t}}\alpha_2$

$R = \lambda v_1(t) + e^{\lambda t}\alpha_2 = \lambda \cancel{e^{\lambda t}}\alpha_1 + \lambda t \cancel{e^{\lambda t}}\alpha_2 + \cancel{e^{\lambda t}}\alpha_2$

Also ✓

$$\Rightarrow v(t) = \begin{pmatrix} e^{\lambda t}\alpha_1 + te^{\lambda t}\alpha_2 \\ e^{\lambda t}\alpha_2 \end{pmatrix} \Rightarrow$$

Mit $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ gilt

$$v(t) := Z(t)\alpha = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ \emptyset & e^{\lambda t} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}.$$

D.h. $Z(t)$ ist die FM des entk. Systems.

Im Falle dass A diagonal. mit λ_1, λ_2 hat $Z(t)$ die Form

$$Z(t) = \begin{pmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{pmatrix}.$$

Notation:

$$\begin{cases} v(t) := Z(t)\alpha \\ y(t) := Y(t)\alpha \end{cases}$$

Wir berechnen $y(t)$:

$$y(t) = X v(t) = \underbrace{X Z(t)}_{Y(t)}\alpha = \begin{pmatrix} | & | \\ x & h \\ | & | \end{pmatrix} \begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{pmatrix} \alpha$$

$$\Downarrow$$

$$y(t) = Y(t)\alpha = \left(\begin{array}{c|c} | & | \\ e^{\lambda t}x & te^{\lambda t}x + e^{\lambda t}h \\ | & | \end{array}\right)\alpha =$$

$$= \left(\begin{array}{c|c} | & | \\ e^{\lambda t}x & e^{\lambda t}(h + tx) \\ | & | \end{array}\right)\alpha$$

Im Falle, dass A diagonal. mit λ_1, λ_2 hat $Y(t)$ die Form

$$Y(t) = \left(\begin{array}{c|c} e^{\lambda_1 t} x^{(1)} & e^{\lambda_1 t} x^{(2)} \end{array} \right),$$

wobei $x^{(1)}$ und $x^{(2)}$ die EV sind;

$$A x^{(1)} = \lambda_1 x^{(1)}, \quad A x^{(2)} = \lambda_2 x^{(2)}.$$

Beispiel:

$n=4$ $\quad \dot{y} - Ay = \emptyset$ mit $A = XJX^{-1}$ und

$$X = \begin{pmatrix} x, & h^{(1)} & h^{(2)} & h^{(3)} \end{pmatrix}, \quad J = \begin{pmatrix} \lambda & 1 & & \\ & \lambda & 1 & \\ & & \lambda & 1 \\ & & & \lambda \end{pmatrix},$$

$$\Rightarrow y(t) = \alpha_1 e^{\lambda t} x + \alpha_2 e^{\lambda t} \left(h^{(1)} + t x \right) +$$

$$+ \alpha_3 e^{\lambda t} \left(h^{(2)} + h^{(1)} t + x \frac{t^2}{2!} \right) +$$

$$+ \alpha_4 e^{\lambda t} \left(h^{(3)} + h^{(2)} t + h^{(1)} \frac{t^2}{2!} + x \frac{t^3}{3!} \right)$$

$\underline{n=2}$ $A \in \mathbb{R}^{2\times 2}$, Komplexe EW:

$$\begin{cases} \lambda_1 = \alpha + i\beta, \ \alpha,\beta \in \mathbb{R}: x^{(1)} = u + iv \\ \qquad\qquad\qquad\qquad\qquad\qquad u, v \in \mathbb{R}^2 \\ \lambda_2 = \alpha - i\beta \qquad\qquad\quad : x^{(2)} = u - iv \end{cases}$$

Finde y reellwertig von

$\dot{y} - Ay = \emptyset \Rightarrow$

$$\boxed{y(t) = \alpha_1 \underbrace{e^{\lambda_1 t} x^{(1)}}_{y^{(1)}} + \alpha_2 \underbrace{e^{\lambda_2 t} x^{(2)}}_{y^{(2)}}; \ \alpha_1 \alpha_2 \in \mathbb{C}}$$

$$\Rightarrow y^{(1)} = e^{\lambda_1 t} x^{(1)} = e^{(\alpha + i\beta)t}(u + iv) =$$

$$= e^{\alpha t}(\cos\beta t + i \sin\beta t)(u + iv) =$$

$$= e^{\alpha t}(u\cos\beta t - v\sin\beta t) + i e^{\alpha t}(v\cos\beta t + u\sin\beta t)$$

$$= \boxed{e^{\alpha t}(u\cos\beta t - v\sin\beta t) + i e^{\alpha t}(v\cos\beta t + u\sin\beta t)}$$

$$\Rightarrow y^{(2)} = e^{\lambda_2 t} x^{(2)} = e^{(\alpha - i\beta)t}(u - iv) =$$

$$= e^{\alpha t}(\cos\beta t - i\sin\beta t)(u - iv) =$$

$$= \boxed{e^{\alpha t}(u\cos\beta t - v\sin\beta t) \pm i e^{\alpha t}(v\cos\beta t + u\sin\beta t)}$$

$y^{(1)}$ und $y^{(2)}$ sind Lösungen, d.h. jede LK von $y^{(1)}$ und $y^{(2)}$ ist auch eine Lösg.

Wähle: $\dfrac{y^{(1)} + y^{(2)}}{2}(t) = e^{\alpha t}(u\cos\beta t - v\sin\beta t)$

Wähle: $\dfrac{y^{(1)} - y^{(2)}}{2i} = e^{\alpha t}(v\cos\beta t + u\sin\beta t)$

$$\Rightarrow \boxed{y(t) = \alpha_1 e^{\alpha t}(u\cos\beta t - v\sin\beta t) + \alpha_2 \cdot e^{\alpha t}(v\cos\beta t + u\sin\beta t) \qquad \underline{\underline{\alpha_1, \alpha_2 \in \mathbb{R}}}}$$

Beispiel $\dot{y}(t) - Ay(t) = \emptyset$, $A = \begin{pmatrix} 1 & -2 \\ 2 & +1 \end{pmatrix}$

1. Schritt $\det(A - \lambda I) = \emptyset \iff$

$$\det \begin{pmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{pmatrix} = \emptyset \iff$$

$$p(\lambda) = (1-\lambda)^2 + 4 = 0 \iff 1 - 2\lambda + \lambda^2 + 4 = \emptyset$$

$$\iff \boxed{\lambda^2 - 2\lambda + 5 = \emptyset}$$

$$\lambda_1, \lambda_2 = \frac{2 \pm \sqrt{4 - 4 \cdot 5}}{2} = \frac{2 \pm \sqrt{-16}}{2}$$

$$= \frac{2 \pm 4i}{2}$$

$$\boxed{\lambda_1 = 1 + 2i, \qquad \lambda_2 = 1 - 2i}$$

2. Schritt $\lambda_1 = 1 + 2i$; $m_1 = 1$, $g_1 = 1 \Rightarrow$

$$(A - \lambda_1 I) x^{(1)} = \emptyset \iff$$

$$\begin{pmatrix} 1-1-2i & -2 \\ 2 & 1-1-2i \end{pmatrix} x^{(1)} = \emptyset \iff$$

$$\begin{pmatrix} -2i & -2 \\ \cancel{2} & \cancel{-2i} \end{pmatrix} x^{(1)} = \emptyset \Rightarrow$$

$$-\cancel{2}ix_1 - \cancel{2}x_2 = \emptyset \Leftrightarrow \quad \Big\| \; x^{(1)} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$i x_1 + x_2 = \emptyset.$$

Wähle $\underline{\underline{x_2 = i}} \Rightarrow \cancel{i}x_1 + \cancel{i} = \emptyset \Rightarrow \underline{\underline{x_1 = -1}}$

$$\Rightarrow \begin{cases} x^{(1)} = \begin{pmatrix} -1 \\ i \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ x^{(2)} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \end{cases}$$

Also,

$$\begin{cases} \lambda := \alpha + i\beta = 1 + 2i \; ; \; \alpha = 1, \beta = 2 \\ x^{(1)} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ 1 \end{pmatrix} =: u + iv, \; u = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{cases}$$

$\Rightarrow$

$$y(t) = \alpha_1 e^{\alpha t} (u \cos\beta t - v \sin\beta t) +$$
$$+ \alpha_2 e^{\alpha t} (v \cos\beta t + u \sin\beta t) \; ; \; \alpha_1, \alpha_2 \in \mathbb{R}$$

$$\Rightarrow y(t) = \alpha_1 e^{t} \left(\begin{pmatrix} -1 \\ 0 \end{pmatrix} \cos 2t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin 2t \right) +$$

$$+ \alpha_2 e^{t} \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos 2t + \begin{pmatrix} -1 \\ 0 \end{pmatrix} \sin 2t \right)$$

$$= \alpha_1 e^{t} \begin{pmatrix} -\cos 2t \\ -\sin 2t \end{pmatrix} + \alpha_2 e^{t} \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix}$$

$$\Rightarrow \underset{FM}{Y(t)} = e^{t} \left(\begin{array}{c|c} -\cos 2t & -\sin 2t \\ -\sin 2t & \cos 2t \end{array} \right) \text{ regulär } \forall t$$

<u>Beispiel</u>
$$\begin{cases} n = 5, \quad \lambda_1, \lambda_2 \\ m_1 = 2, \quad m_2 = 3 \\ g_1 = 1, \quad g_2 = 2 \end{cases}$$

$$J = \begin{pmatrix} \boxed{\begin{matrix} \lambda_1 & 1 \\ & \lambda_1 \end{matrix}} & & \\ & \boxed{\lambda_2} & \\ & & \boxed{\begin{matrix} \lambda_2 & 1 \\ & \lambda_2 \end{matrix}} \end{pmatrix}$$

$$X = \left(\boxed{\begin{matrix} | & | \\ x^{(1)} & h_1^{(1)} \\ | & | \end{matrix}} \; \boxed{\begin{matrix} | \\ x^{(2)} \\ | \end{matrix}} \; \boxed{\begin{matrix} | & | \\ x^{(3)} & h_1^{(3)} \\ | & | \end{matrix}} \right)$$

$$y(t) = \underbrace{\alpha_1 e^{\lambda_1 t} x^{(1)} + \alpha_2 e^{\lambda_1 t}\left(h^{(1)} + t\,x^{(1)}\right)}$$

$$+ \underbrace{\alpha_3 e^{\lambda_2 t} x^{(2)}} + \underbrace{\alpha_4 e^{\lambda_2 t} x^{(3)} +}$$

$$\underbrace{+\alpha_5 e^{\lambda_2 t}\left(h^{(3)} + t x^{(3)}\right)}$$

Löse $\quad \dot{y}(t) - A y(t) = f(t).$

$\alpha \in \mathbb{R}^n$, $y(t) = Y(t)\alpha$ ... allg. Lösg der hom. Gl. ($f(t) \equiv \emptyset$)

$$\boxed{y_p(t) := Y(t)\alpha(t) \ \dots \ \text{Variat. der Konst.}}$$

↑ muß $\dot{y} - Ay = f$ erfüllen.

D.h.

$$\dot{y}_p(t) = \dot{Y}(t)\alpha(t) + Y(t)\dot{\alpha}(t)$$

Einsetzen in die DGl.

$$\Rightarrow \dot{Y}(t)\alpha(t) + \underbrace{Y(t)\dot{\alpha}(t) - A\,Y(t)\alpha(t)} = f(t)$$

$$\Rightarrow \underbrace{\left(\dot{Y}(t) - A Y(t)\right)}_{=\emptyset}\alpha(t) + Y(t)\dot{\alpha}(t) = f(t)$$

$\Downarrow$

$$\Rightarrow \quad Y(t)\dot{\alpha}(t) = f(t) \quad \Longleftrightarrow$$

$Y(t)$ regulär $\forall t$

Beweis von $\dot{Y}(t) - AY(t) = 0$:

$$Y(t) = \begin{pmatrix} | & | & & | \\ y_1(t) & y_2(t) & \cdots & y_n(t) \\ | & | & & | \end{pmatrix}.$$ Dabei sind

y_i lin.-unabh. Lösungen des hom. Systems $\Rightarrow \dot{y}_i(t) - Ay_i(t) = \emptyset \quad \forall i$

$$\Rightarrow \dot{Y}(t) - AY(t) = \begin{pmatrix} | & | & & | \\ \dot{y}_1 & \dot{y}_2 & \cdots & \dot{y}_n \\ | & | & & | \end{pmatrix} - A\begin{pmatrix} | & & | \\ y_1 & \cdots & y_n \\ | & & | \end{pmatrix}$$

$$= \begin{pmatrix} | & | & & | \\ \dot{y}_1 - Ay_1 & \dot{y}_2 - Ay_2 & \cdots & \dot{y}_n - Ay_n \\ | & | & & | \end{pmatrix} = \emptyset$$

$$\Rightarrow \quad \dot{\alpha}(t) = Y^{-1}(t)f(t)$$

$$\boxed{\alpha(t) = \int_{t_0}^{t} Y^{-1}(s)f(s)\,ds + \alpha(t_0)}$$

Also $y_p(t) = Y(t)\alpha(t) =$

$$= Y(t)\int_{t_0}^{t} Y^{-1}(s) f(s)\,ds + Y(t)\alpha(t_0)$$

Wähle $\alpha(t_0) = \emptyset$

$$\Rightarrow \quad y_p(t) = Y(t)\int_{t_0}^{t} Y^{-1}(s) f(s)\,ds$$

$$\Rightarrow \quad \boxed{y(t) = Y(t)\alpha + Y(t)\int_{t_0}^{t} Y^{-1}(s) f(s)\,ds}$$

<u>Beispiel 7.4</u>

<u>Löse das AWP</u>

$$\dot{y}(t) - Ay(t) = \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ -2 \end{pmatrix}}_{f_1(t)} + \underbrace{e^t\begin{pmatrix} 2 \\ 0 \end{pmatrix}}_{f_2(t)}$$

$$A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}, \; y(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

Wir wissen:

$$y^h(t) = \alpha_1 e^t \begin{pmatrix} -\cos 2t \\ -\sin 2t \end{pmatrix} + \alpha_2 e^t \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix}.$$

Berechnung der Partikulärlösg.

Ansatz: $\begin{cases} y_1^p(t) := a + tb, & a, b \in \mathbb{R}^2 \\ y_2^p(t) = e^t c, & c \in \mathbb{R}^2 \end{cases}$

$\underbrace{\dot{y}_1(t) = b \Rightarrow \text{DGl.} \Rightarrow}$

$$\Rightarrow \quad b - A(a + tb) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \end{pmatrix} \quad \forall t$$

$$\Leftrightarrow \quad (b - Aa) + t(-Ab) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} t: & +Ab = \begin{pmatrix} +1 \\ +2 \end{pmatrix} \Leftrightarrow \underbrace{\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}} \quad \forall t \\ t^0: & b - Aa = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{cases}$$

$$\Downarrow$$

$$\underline{\underline{b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = Aa = \underbrace{\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}}_{\text{regulär}} a \Rightarrow \underline{\underline{a = \vec{0}}}$$

$$\Rightarrow \boxed{y_1^p(t) = t \begin{pmatrix} 1 \\ 0 \end{pmatrix}}$$

$\underbrace{\dot{y}_2^p(t) = e^t c \Rightarrow \text{DGl.} \Rightarrow}$

$$\Rightarrow \quad e^t c - A e^t c = e^t \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad \forall t$$

$$\Rightarrow \cancel{e^t}(I - A)c = \cancel{e^t}\begin{pmatrix}2\\0\end{pmatrix} \quad \forall t$$

$$\Rightarrow \left(\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} - \begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}\right)c = \begin{pmatrix}2\\0\end{pmatrix}$$

$$\Rightarrow \boxed{\begin{pmatrix}0 & 2\\-2 & 0\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}}$$

$$\Leftrightarrow \begin{cases} \cancel{2}c_2 = \cancel{2} & \Rightarrow c_2 = 1 \\ -2c_1 = 0 & \Rightarrow c_1 = 0 \end{cases}$$

$$\Rightarrow \boxed{y_2^p(t) = e^t\begin{pmatrix}0\\1\end{pmatrix}}$$

Also

$$y(t) = y^h(t) + y_1^{(p)}(t) + y_2^{(p)}(t) =$$

$$= \alpha_1 e^t\begin{pmatrix}-\cos 2t\\-\sin 2t\end{pmatrix} + \alpha_2 e^t\begin{pmatrix}-\sin 2t\\\cos 2t\end{pmatrix} + t\begin{pmatrix}1\\0\end{pmatrix} + e^t\begin{pmatrix}0\\1\end{pmatrix}$$

$$y(0) = \begin{pmatrix}3\\0\end{pmatrix} = \begin{pmatrix}-\alpha_1\\0\end{pmatrix} + \begin{pmatrix}0\\\alpha_2\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}$$

$$\Rightarrow \boxed{\alpha_1 = -3 \quad \alpha_2 = -1}$$

$$\Rightarrow \begin{cases} \dot{y}_1(t) \boxed{- y_1(t) + 2 y_2(t)} + f_1(t) = \emptyset \\ \dot{y}_2(t) \boxed{- 2 y_1(t) - y_2(t)} + f_2(t) = \emptyset \end{cases} \Rightarrow$$

$$\Longleftrightarrow \quad \dot{y}(t) - A y(t) + f(t) = \emptyset$$

$$\text{mit} \quad A = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \Rightarrow$$

Wegen

$$-Ay = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

Also

$$y(t) = -3e^{t} \begin{pmatrix} -\cos 2t \\ -\sin 2t \end{pmatrix} - e^{t} \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3e^{t} \cos 2t + e^{t} \sin 2t + t \\ 3e^{t} \sin 2t - e^{t} \cos 2t + e^{t} \end{pmatrix}.$$