

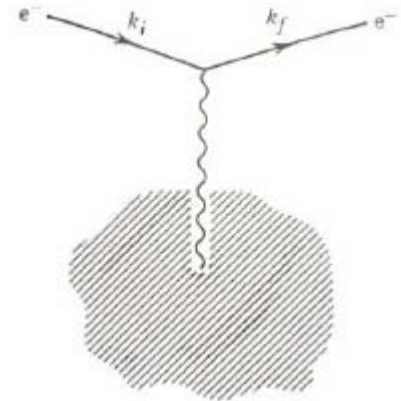
# Quarks, Gluons, QCD

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- Quarks: from a concept of classification to physics reality
- Deep inelastic electron scattering
  - Pointlike constituents: 'partons'
  - Quantitative analysis: partons have spin  $\frac{1}{2}$  and fractional charge
- $e^+e^-$  annihilation:
  - Number of quarks; color charge of quarks
  - Discovery of gluons
- QCD Lagrangian
  - Difference to QED
  - Quark-Gluon Plasma

# Probing the size of the proton

- Probing the charge distribution, shown in figure



- Approach; measure the angular distribution of electrons and compare to pointlike distribution

- $\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(q)|^2$  with  $q = k_i - k_f$ ;  $|F(q)|$ ...Form factor

- Example: scattering of unpolarized electrons from static charge distribution  $Ze\rho(\vec{x})$

- For a static target:  $F(\vec{q}) = \int \rho(\vec{x}) e^{i\vec{q}\cdot\vec{x}} d^3x$  ... Fourier transform of charge distribution

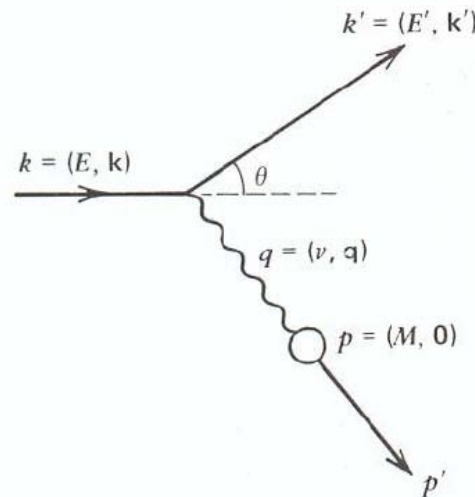
- Form factor is Fourier transform of charge distribution

- Lorentz invariant four-momentum transfer

$$q^2 = (E - E')^2 - (\vec{p} - \vec{p}')^2 \cong -4 E E' \sin^2(\theta/2)$$

# $e^- \mu^- \rightarrow e^- \mu^-$

- Reaction is relevant for understanding lepton scattering on constituents
- Scattering cross section in Lab frame (muon at rest, mass M)



$$\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left( \cos^2 \theta/2 - \frac{q^2}{2M^2} \sin^2 \theta/2 \right)$$

- Scattering cross section of electron on spin  $\frac{1}{2}$  particle
- Electron beam used to study dimension and internal structure of protons

# Charge distribution of proton

- For  $|q|$  small; (small energy transfer, large 'equivalent' wavelength of electron)

$$F(\vec{q}) = \int \left( 1 + i\vec{q} \cdot \vec{x} - \frac{(\vec{q} \cdot \vec{x})^2}{2} + \dots \right) \rho(\vec{x}) d^3x = 1 - \frac{1}{6} |\vec{q}|^2 \langle r^2 \rangle$$

assuming that charge distribution is spherically symmetric

- Low  $|q|$ , i.e. small angle scattering measures the mean square charge radius
- Cannot directly be applied to protons
  - Need to consider magnetic moment; proton not static, will recoil
- Reference point-like cross-section is same as  $e\mu$  scattering with  $M_p$

$$\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left( A \cos^2 \theta/2 - B \frac{q^2}{2M^2} \sin^2 \theta/2 \right)$$

where  $A, B = 1$  for point-like proton;  $E/E'$  from proton recoil

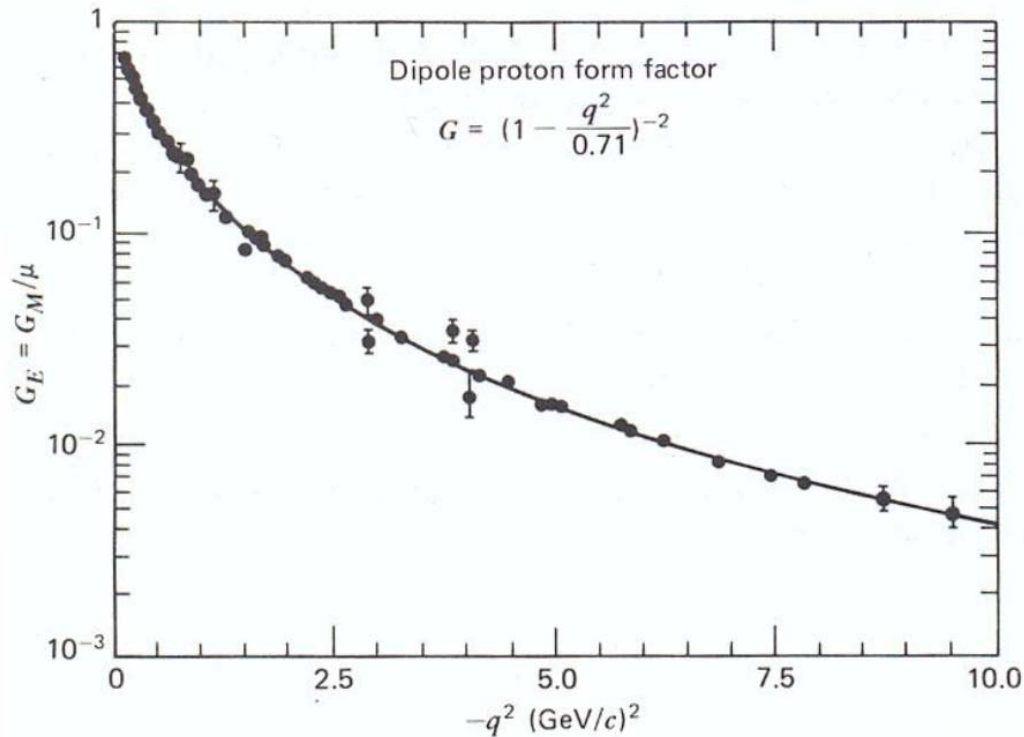
# Charge distribution of proton

- Generalizing to extended source, one obtains two form factors (electric and magnetic ) with  $\kappa$  being the anomalous magnetic moment with the result

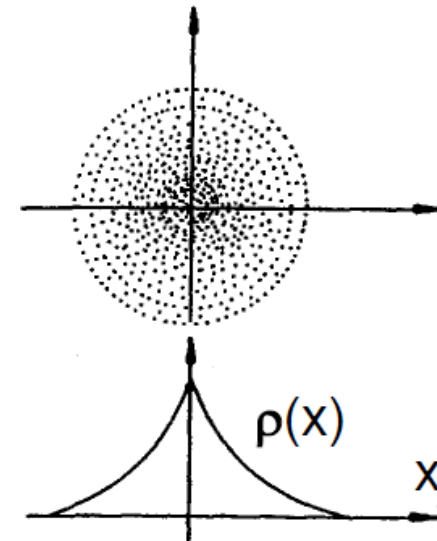
- $$A = \left( F_1^2 - \frac{\kappa^2}{4M^2} F_2^2 \right) , \quad B = -\frac{q^2}{2M^2} (F_1 + \kappa F_2)^2$$

- ‘Rosenbluth’ formula; the two form factors  $F_{1,2}(q^2)$  summarize the structure of the proton; determined experimentally; formula reduces to pointlike formula for  $\kappa=0$  and  $F_1(q^2) = 1$
- In practise  $G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2$        $G_M = F_1 + \kappa F_2$
- For protons:  $\langle r^2 \rangle = (0.81 * 10^{-13} \text{ cm})^2$
- Nobel prize for Hofstaedter in 1961

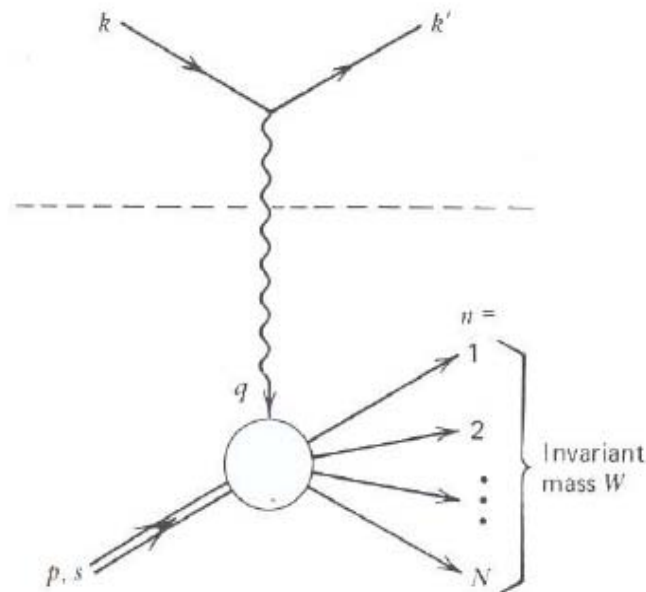
# Proton form factor versus $q^2$



Fourier transform of this  
Form factor is exponential  
Charge distribution  
 $\rho(r) = \rho_0 \exp(-q_0 / r)$

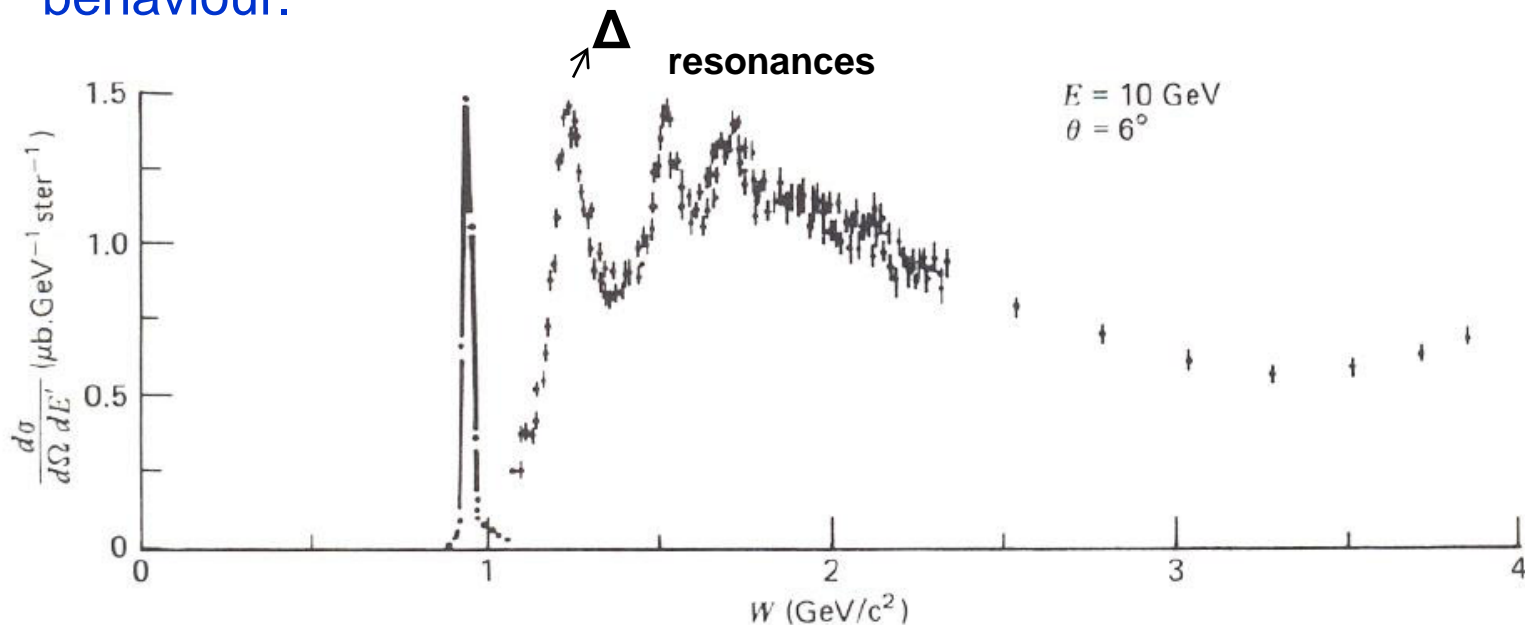


- Probing the internal structure of the proton
  - Increase the momentum transfer  $q^2$  of the photon, equivalent to photons of shorter wavelength
  - However, if proton is composite object, it will get excited, break up under large momentum transfer, producing system of particles with invariant mass  $W$



# The $ep \rightarrow eX$ cross section

- The  $ep \rightarrow eX$  cross section as a function of the invariant mass of the particle system produced. The peak at  $W \approx M$  corresponds to scattering which does not breakup the proton; the peaks at higher  $W$  correspond to excited states of the proton; beyond the resonances multiparticle states with large invariant mass result in a smooth behaviour.

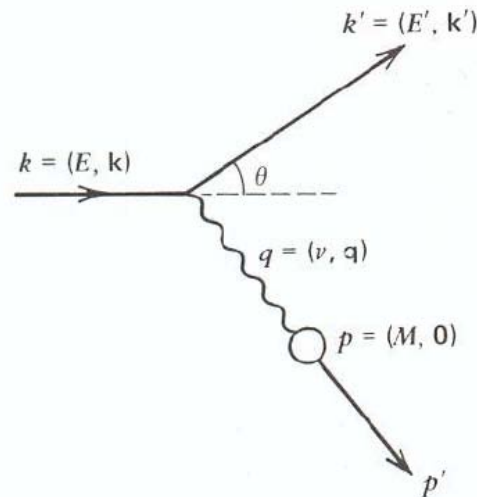


(elastic peak at  $W=M_p$  is reduced by factor 8.5)



# $e^- \mu^- \rightarrow e^- \mu^-$

- Reaction is relevant for understanding lepton scattering on constituents
- Scattering cross section in Lab frame (muon at rest, mass  $M$ )



$$\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left( \cos^2 \theta/2 - \frac{q^2}{2M^2} \sin^2 \theta/2 \right)$$

- Scattering cross section of electron on spin  $\frac{1}{2}$  particle
- Electron beam used to study dimension and internal structure of protons

# Deep inelastic scattering

- Generalization of the inelastic scattering process follows the formalism for  $e^- \mu^- \rightarrow e^- \mu^-$ , but requires a more complicated description of the proton interaction, with two independent variables
  - $q^2$ ,  $\nu = \mathbf{p} \cdot \mathbf{q} / M$ ,  $q$ ...four-momentum of virtual photon;  
 $M$  ..proton mass;
  - or alternatively
  - $x = -q^2 / 2 \mathbf{p} \cdot \mathbf{q}$ ;  $y = \mathbf{p} \cdot \mathbf{q} / \mathbf{p} \cdot \mathbf{k}$
  - Invariant mass of final hadronic system

$$W^2 = (\mathbf{p} + \mathbf{q})^2 = M^2 + 2\nu M + q^2$$

- Giving the final result

$$\left( \frac{d\sigma}{dE' d\Omega} \right)_{lab} = \frac{4\alpha^2 E'^2}{q^4} \left( W_2(\nu, q^2) \cos^2 \theta / 2 + 2W_1(\nu, q^2) \sin^2 \theta / 2 \right)$$

with  $W_1$  and  $W_2$  to be determined experimentally... see later

# Summary: electron scattering

- The differential cross section for  $e\mu \rightarrow e\mu$ ,  $ep \rightarrow ep$  (elastic) and  $ep \rightarrow eX$  can be written as

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{lab} = \frac{4\alpha^2 E'^2}{q^4} \{...\}$$

- For  $e\mu \rightarrow e\mu$

$$\{...\} = \left( \cos^2 \theta / 2 - \frac{q^2}{2m^2} \sin^2 \theta / 2 \right) \delta\left(\nu + \frac{q^2}{2m}\right)$$

- For  $ep \rightarrow ep$  (elastic)

$$\{...\} = \left( \frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \theta / 2 + 2\tau G_M^2 \sin^2 \theta / 2 \right) \delta\left(\nu + \frac{q^2}{2M}\right)$$

- Integration over  $\delta$ -function gives

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2 \sin^4 \theta / 2} \frac{E'}{E} (...)$$

- For  $ep \rightarrow eX$  very similar; see expression on previous slide

# Studying the sub-structure of the proton

- The formalism developed for deep inelastic scattering  $ep \rightarrow eX$  can be applied to the special case of probing a possible proton sub-structure
  - Using sufficiently small wavelength (i.e. sufficiently large  $q^2$ ) it is possible to resolve a possible substructure, i.e. constituents
  - The breaking-up of the proton is described by the inelastic form factors  $W_1$  and  $W_2$
  - The scattering formalism is applied to electron scattering on the constituents, assuming certain properties



# Probing the proton composition

- Assuming **pointlike** constituents ('partons') with **spin**  $\frac{1}{2}$ , the scattering cross section is related to  $e\nu$ -scattering with (for convenience  $Q^2 = -q^2$ )

$$2W_1^{point}(\nu, Q^2) = \frac{Q^2}{2m^2} \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$W_2^{point}(\nu, Q^2) = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

- $m$  is the mass of the parton (or quark); pointlike: structureless Dirac particle
- Using the identity  $\delta(x/a) = a \delta(x)$  one finds

$$2mW_1^{point}(\nu, Q^2) = \frac{Q^2}{2m\nu} \delta\left(\nu - \frac{Q^2}{2m\nu}\right) \quad \nu W_2^{point}(\nu, Q^2) = \delta\left(\nu - \frac{Q^2}{2m\nu}\right)$$

- With the intriguing result that these functions depend only on the ratio  $Q^2/2m\nu$  and not on  $Q^2$  and  $\nu$  independently

# Probing the proton composition

- Summarizing and replacing the parton mass scale with the proton mass scale  $M$

$$MW_1^{point}(\nu, Q^2) \rightarrow F_1(\omega) \quad \nu W_2(\nu, Q^2) \rightarrow F_2(\omega)$$

for large  $Q^2$  and  $\omega = 2M\nu / Q^2$ ; at a given  $\omega$ , the structure functions **are measured** to be independent of  $Q^2$

- Inelastic structure functions are independent on  $Q^2$  -> constituents are pointlike and quasi-free (inside the proton)

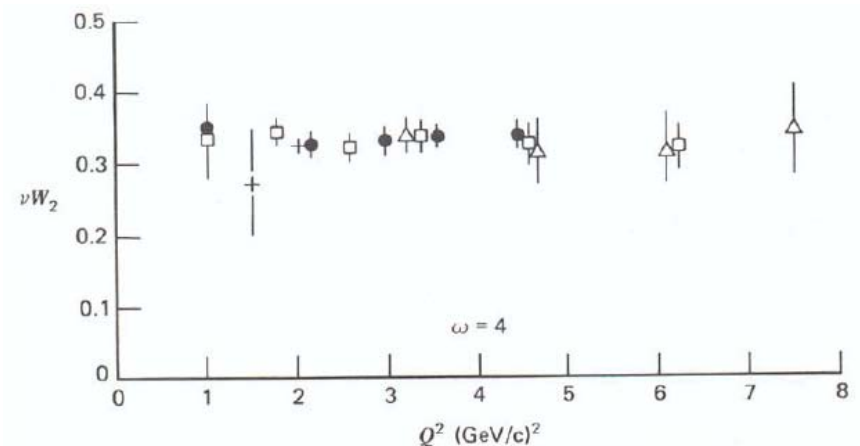
- One experimental example

- Structure function = Fourier

Transform of charge distribution  $\rightarrow$

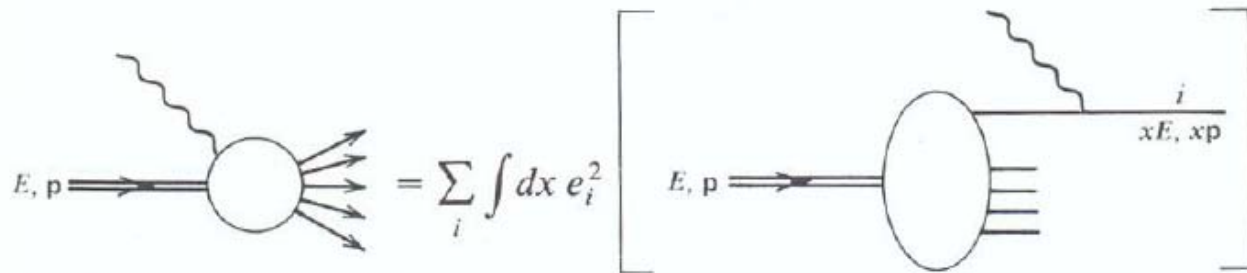
St.F. is constant  $\rightarrow$  charge distrib.

is pointlike!

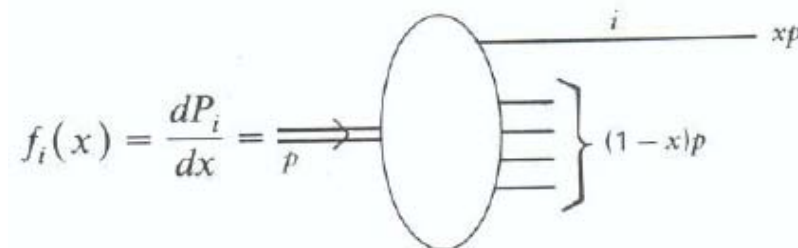


# What are the properties of the 'partons' ?

- Partons are spin  $\frac{1}{2}$ , electrically charged pointlike particles



- This picture recognizes that there are various partons in the proton: e.g. u, d quarks with different charges; uncharged gluons, with which the photon does not react; they carry different fraction  $x$  of the parent proton's momentum and energy ->
- Parton momentum distribution



# Parton momentum distribution functions

- $f_i(x)$  gives probability that parton  $i$  carries fraction  $x$  of the proton's momentum  $p$ ; all the fractions have to add up to 1

$$\sum \int dx x f_i(x) = 1$$

- Which leads to the following expressions for the structure functions

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$MW_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x)$$

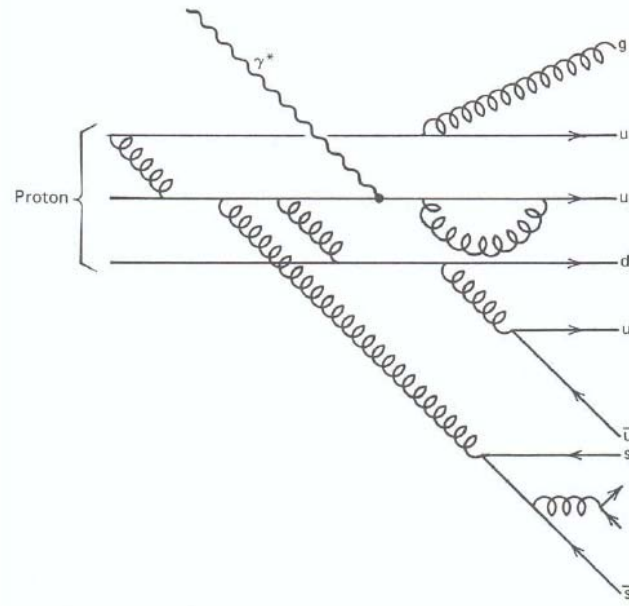
with  $x = 1/\omega = Q^2/2M\nu$ , only dependent on  $x$

- The momentum fraction is found to be identical to the kinematical variable  $x$  of the virtual photon: the virtual photon must have the right value of  $x$  to be absorbed by the parton with momentum fraction  $x$



# Looking at quarks inside the proton

- Proton is composed of the constituent quarks (u,d quarks) (or ‘valence’ quarks), gluons, and quark-antiquark pairs (‘sea’ quarks)



- For the proton

$$\frac{1}{x} F_2^p(x) = \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \bar{s}^p(x)]$$

# Looking at quarks inside the proton

- Six unknown quark structure functions; additional information is provided by measuring electron-deuteron scattering, providing information on the corresponding neutron structure functions

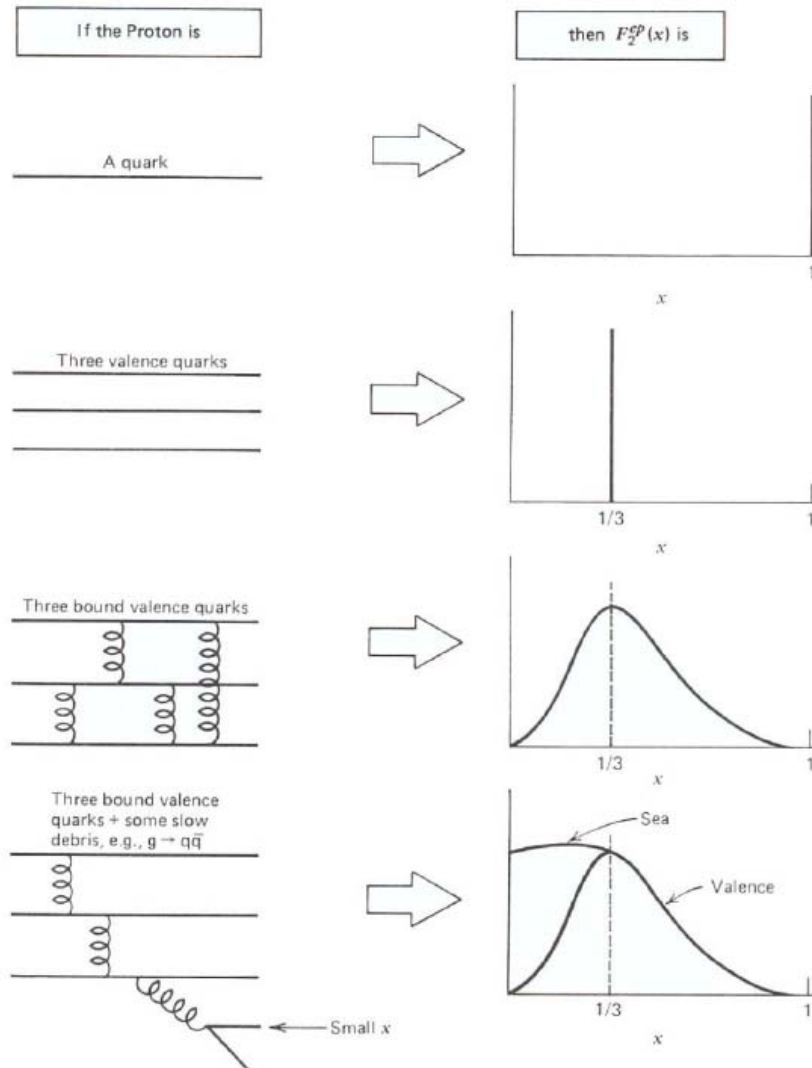
$$\frac{1}{x} F_2^n(x) = \left(\frac{2}{3}\right)^2 [u^n(x) + \bar{u}^n(x)] + \left(\frac{1}{3}\right)^2 [d^n(x) + \bar{d}^n(x)] + \left(\frac{1}{3}\right)^2 [s^n(x) + \bar{s}^n(x)]$$

- Due to isospin invariance their quark content is related
  - There are as many u quarks in the proton as d quarks in the neutron

$$u^p(x) = d^n(x) \equiv u(x) \quad d^p(x) = u^n(x) \equiv d(x) \quad s^p(x) = s^n(x) \equiv s(x)$$

- Additional constraints: quantum numbers of proton must be those of the uud combination
- Measurement of  $F_2(x)$  confirms charge assignment of the u and d quarks

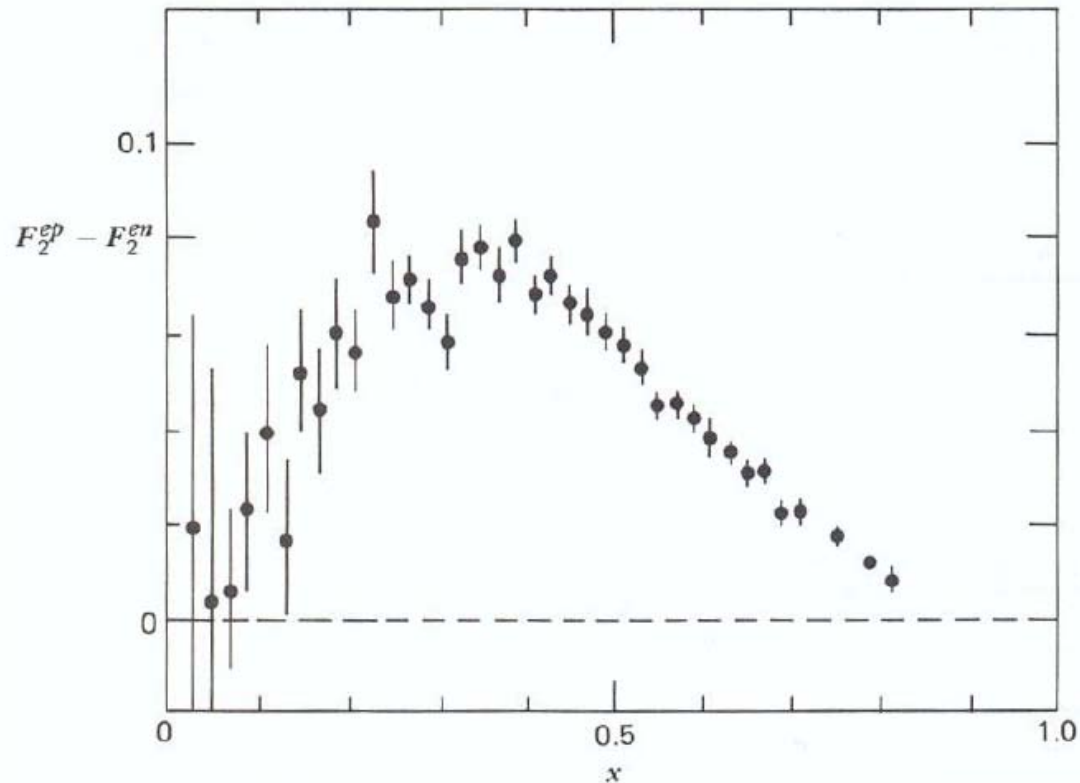
# Conceptual form of the structure functions



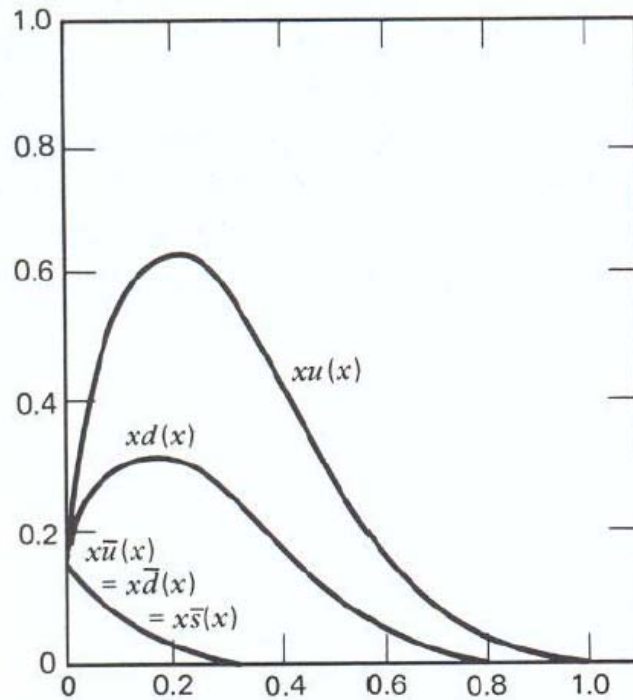
# Valence quark distribution

- From 
$$\frac{1}{x} \left[ F_2^p(x) - F_2^n(x) \right] = \frac{1}{3} \left[ u(x) - d(x) \right]$$

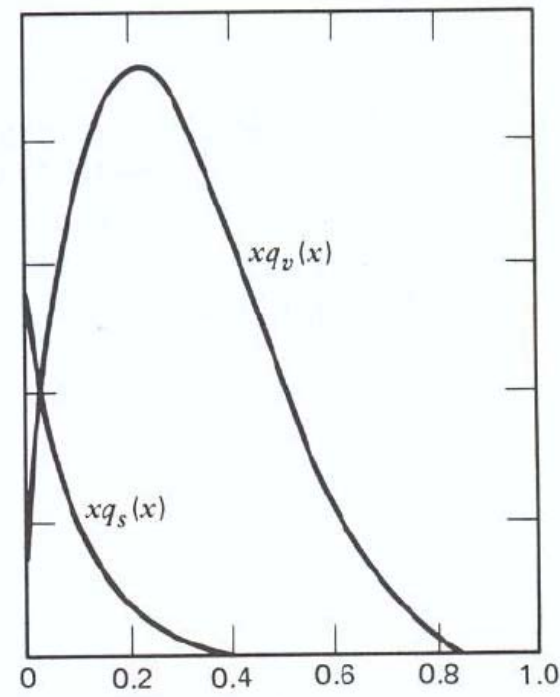
one can directly measure the valence quark distributions



- From the analysis of deep inelastic scattering data

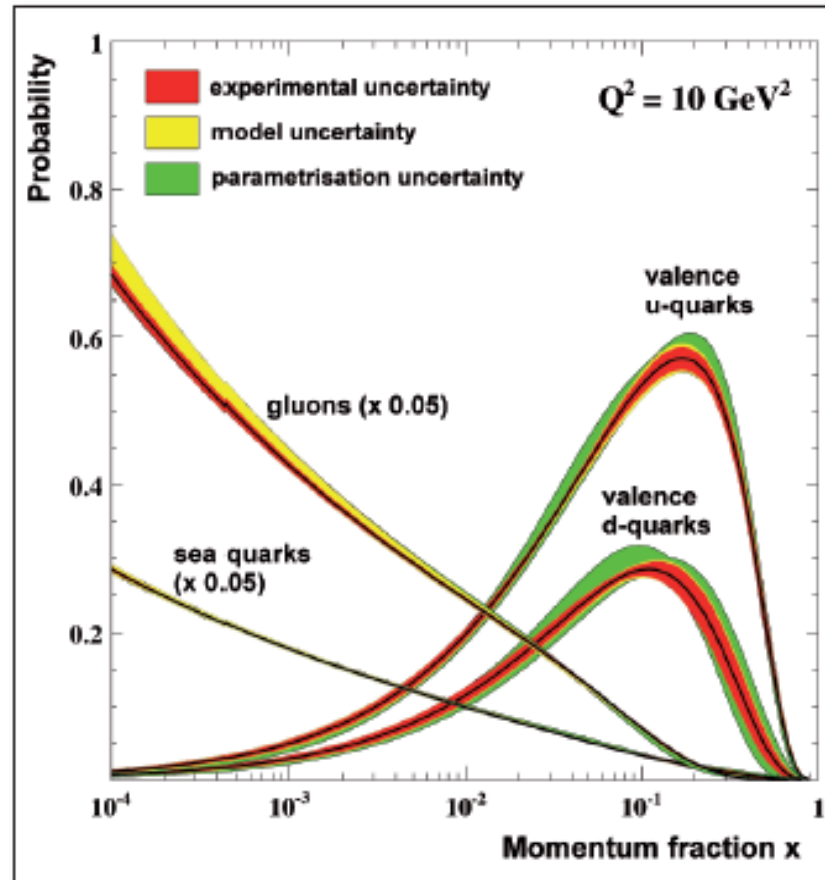


(a)



(b)

# Quark structure functions: 'state of the art'



# Deep inelastic scattering (DIS) experiments at Stanford Linear Accelerator(SLAC)

- Developed in the late 1960's; was at the time one of the largest experimental facilities
- Originally conceived to study elastic scattering-> extension to inelastic scattering met with some scepticism by the Program Committees: what can one learn?
- Established the quark structure
- Nobel prize (1962) for J.I. Friedman, H.W. Kendall and R.E. Taylor for 'structure of the proton'



# Summary: results from DIS

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- From structure functions  $F_2(x, Q^2) \approx F_2(x) \rightarrow$  nucleons are composed from pointlike constituents
- From  $2x F_1(x) = F_2(x) \rightarrow$  constituents have spin  $\frac{1}{2}$
- From experimental data on  $F_2(x)$  for protons and neutrons (supplemented with data from DIS neutrino scattering)  $\rightarrow$  charge assignment for the u and d quarks
- From  $\int F_2(x) dx \rightarrow$  quarks carry approximately 50 % of nucleon momentum; the rest is carried by the gluons; strong evidence for the reality and importance of gluons inside the nucleon
- Quantum numbers of the nucleon can be explained with the quantum number assignment of the quarks



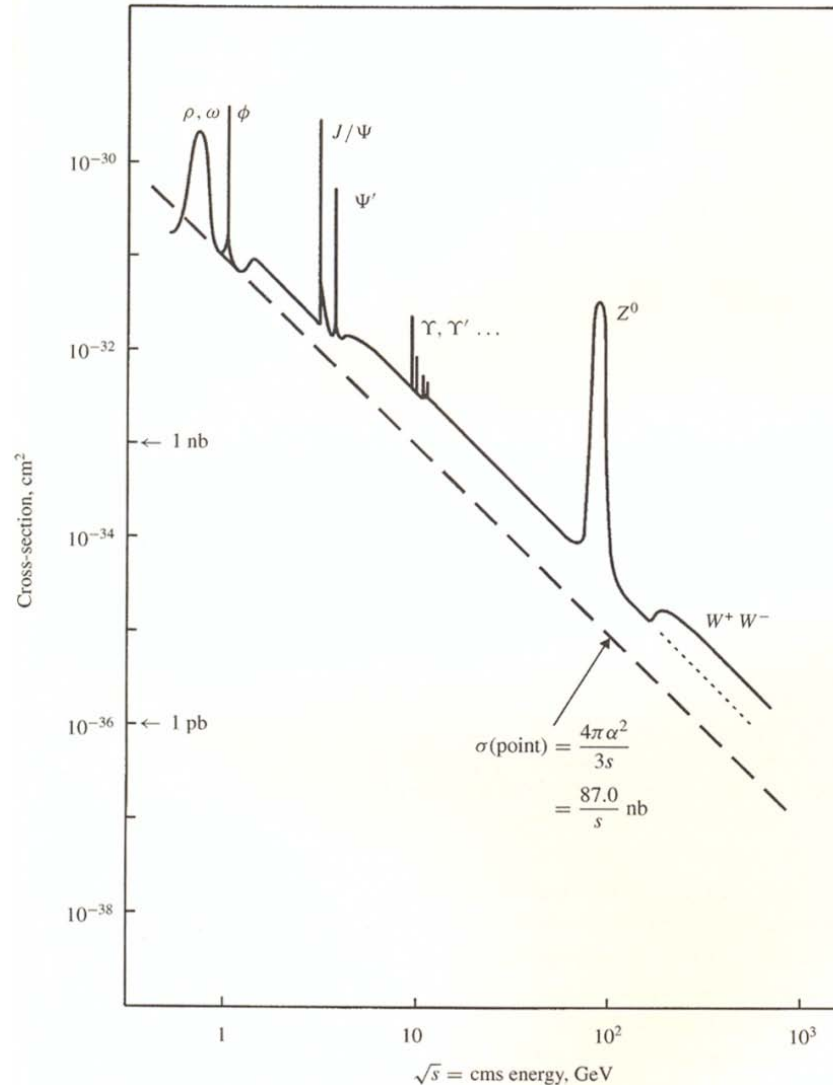
# $e^+e^-$ Annihilations to Hadrons

- $e^+e^- \rightarrow Q\bar{Q} \rightarrow Q \rightarrow \text{hadron jet}, \bar{Q} \rightarrow \text{hadron jet}$



- $\sigma(e^+e^- \rightarrow \text{anything}) \propto \frac{1}{s}$  (as for  $\mu^+\mu^-$  production)  
s...center of mass energy
- Peaks in cross-section are due to boson resonances

# $e^+e^-$ Annihilations to Hadrons vs center of mass energy $s$



# Experimental proof of color charge of quarks

- Measurement of total cross section  $\sigma(e^+e^- \rightarrow \text{hadrons})$  relative to  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$
- Total cross section is obtained by summing over all contributing quarks:

$$\sigma(e^+e^- \rightarrow q\bar{q}) = \sigma(e^+e^- \rightarrow \mu^+\mu^-) \cdot N_C \sum_i q_i^2$$

- $N_C$  is the number of color charges (states)
- the (three) color states of a quark have the same electric charge
- The sum is over all energetically possibly produced quarks
- Measurement of the ratio

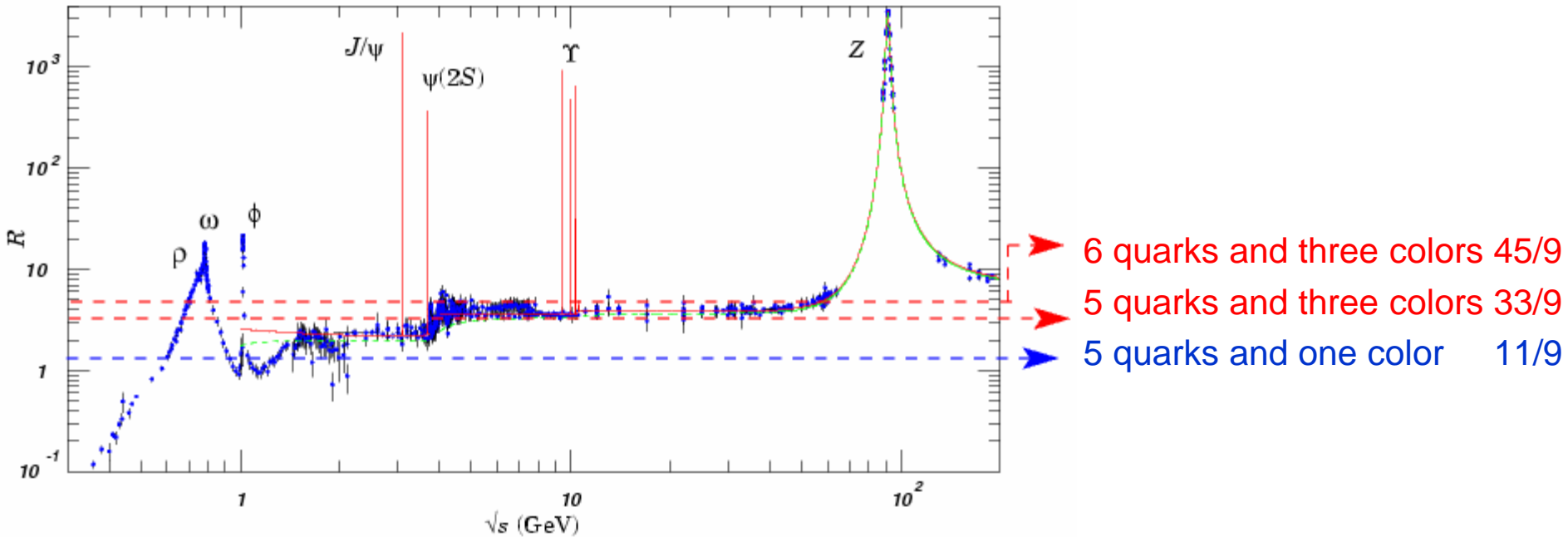
$$R = \sigma(e^+e^- \rightarrow q\bar{q}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-) = N_C \sum_i q_i^2$$

directly determines the number of color states

- Higher order effects ( 3 jets,..) modify R

$$R = R_0 (1 + \alpha_s(Q^2) / \pi + \dots)$$

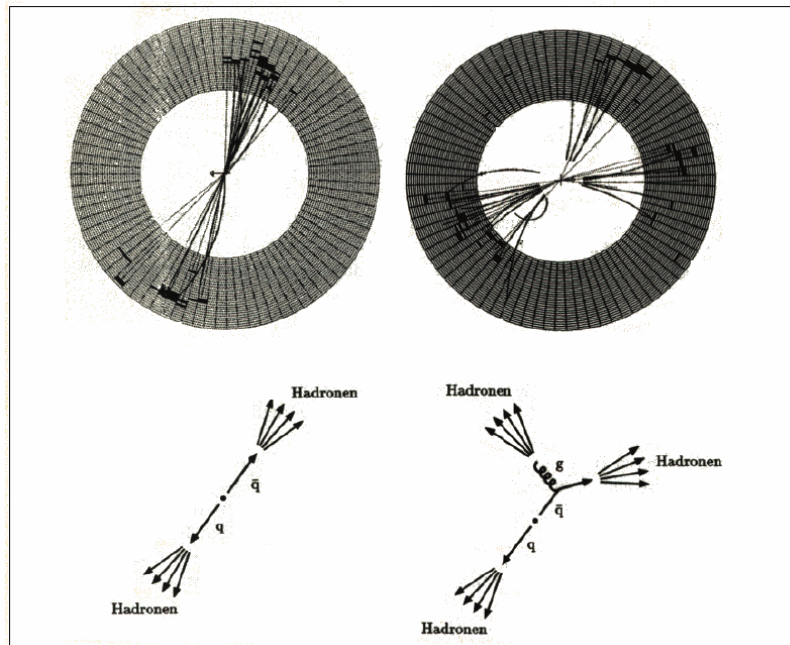
# Measurement of R



$$R = N_c \sum_f z_f^2 = N_c \cdot \left[ \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = N_c \cdot \frac{11}{9}$$

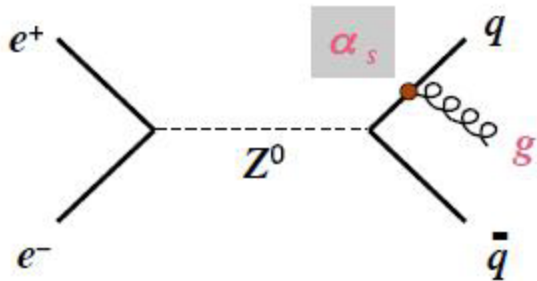
# $e^+e^- \rightarrow$ three Jets

- Discovery of gluons in the observation of  $e^+e^- \rightarrow$  three Jets  $\rightarrow$  quark jet+ antiquark jet + gluon jet
  - gluon is radiated by a quark (or antiquark)
- Independent confirmation in proton-proton collisions (quark-gluon scattering) and DIS (electrons and neutrinos)



# $e^+e^- \rightarrow \text{three Jets}$

- Angular distribution of the gluon jet is sensitive to spin of gluon->
  - Spin of gluon = 1 (vector boson)
- Three-jet events can also be used to determine  $\alpha_s$ : rate of gluon radiation is proportional to  $\alpha_s$



Event display of a 3-jet event at LEP

# Summary: $e^+e^- \rightarrow \text{hadrons}$

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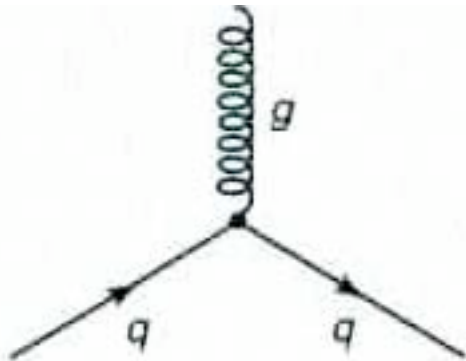
- Measurements are consistent with
  - fractional charge for the quarks and three color states (value of R)
  - Quarks (Antiquarks) can radiate gluons -> gluons have similar reality as quarks
  - Gluon radiation can be quantitatively used to measure  $\alpha_s$  and to determine the spin of the gluon (S=1)

# Theory of Strong Interactions: Quantum Chromodynamics (QCD)

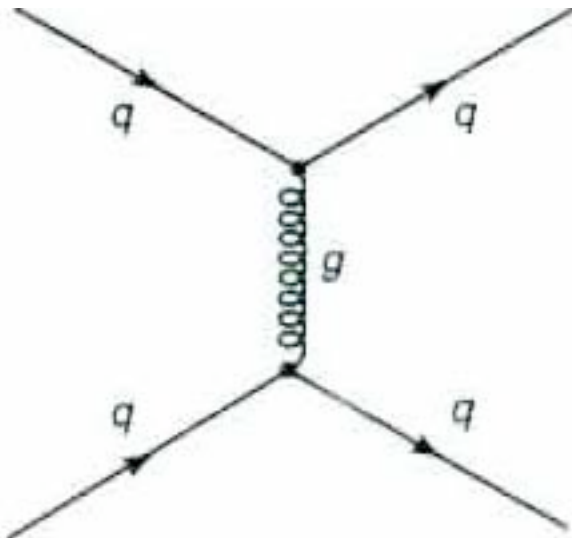
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- Status (approx. 1970)
  - Concept of quarks introduced for classification of hadrons
    - The 'Eightfold Way' by Gell-Mann; similar concept by Zweig
  - Classification needed another ingredient, 'color' charge of quarks
    - Required to avoid problem with Pauli exclusion principle
  - Free quarks were not observed-> are quarks really particles ?
  - DIS showed that proton has a substructure-> partons
    - Detailed experiments confirmed partons to have the properties of quarks( fractional charge, spin  $\frac{1}{2}$ )
  - Quantum Electrodynamics (QED) confirmed with high experimental accuracy-> local gauge invariance as principle for deriving the Lagrangian of particle interactions
    - Experimental tests of nascent electroweak theory contemplated
- Ingredients prepared for attacking the 'hardest' problem: Strong Interactions





- in QCD: color plays the role of charge
- fundamental vertex  
 $q \rightarrow q + \text{gluon } g$
- Analogous to  $e \rightarrow e + \gamma$



- bound state of  $q\bar{q}$
- Scattering of two quarks
- force between two quarks is mediated by the exchange of gluons

# Quantum Chromodynamics QCD: similarities and differences to QED

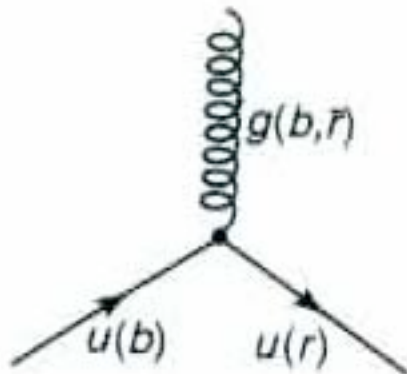
- QED: one type of charge, i.e. *one* number (+, -); photon is neutral
- QCD: *three* kinds of color: red, green, blue

- Fundamental process  $q \rightarrow q + g$

: color of quark (not its flavor may change in strong interactions)

e.g.: blue up-quark  $\Rightarrow$  red up-quark

color is conserved  $\Rightarrow$  gluon carries away the difference



gluons are 'bicolored' with one positive and negative unit (e.g.: one unit of blueness and minus one unit of redness)

$3 \times 3 = 9$  possibilities  $\Rightarrow$  experimentally only 8 different gluons observed; ninth gluon

would be 'color singlet' (color neutral) and therefore observable  $\Rightarrow$  not observed, i.e. does not exist

# QED and QCD: similarities and differences

- QED Lagrangian derived with the requirement of 'local gauge invariance'; gauge group is U(1)

$$\mathcal{L} = [i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi] - \left[ \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} \right] - (q \bar{\psi} \gamma^\mu \psi) A_\mu \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

with  $A_\mu$  a new massless field such that  $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$

- QCD Lagrangian  $\mathcal{L} = i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi$   
with  $\bar{\psi} = (\bar{\psi}_r \bar{\psi}_b \bar{\psi}_g)$ , describes interaction of three (equal mass) color states  $\rightarrow$  require invariance under U(3), with U being a 3x3 matrix which can be written as  $U = e^{i\theta} e^{i\bar{\lambda} \cdot \bar{a}}$ ;  $\bar{\lambda} \cdot \bar{a} = \lambda_1 a_1 + \lambda_2 a_2 + \dots + \lambda_8 a_8$
- Matrix  $e^{i\bar{\lambda} \cdot \bar{a}}$  has determinant 1  $\rightarrow$  belongs to SU(3)  $\rightarrow$  want to derive Lagrangian invariant under local SU(3) invariance
- $\psi \rightarrow S \psi$ , where  $S \equiv e^{iq\bar{\lambda} \cdot \Phi(x)/\hbar c}$ ;  $\Phi \equiv -(\hbar c / g) \bar{a}$ ; g is coupling constant
- Complete QCD Lagrangian

$$\mathcal{L} = [i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi] - \left[ \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} \right] - (q \bar{\psi} \gamma^\mu \bar{\lambda} \psi) A_\mu$$

1<sup>st</sup> term: free quark; 2<sup>nd</sup> term: gluon field; 3<sup>rd</sup> term: quark-gluon interaction

# QCD: Gluon-Gluon coupling



gluons, carrying color, (unlike the electrically neutral photon) may couple to other gluons  
⇒ three and four gluon vertices ⇒ QCD more

complicated (but also richer: allows for more possibilities)

- Coupling constant  $\alpha_s \sim 1 \Rightarrow$  higher order diagrams make significant (sometimes even dominant) contributions: a real problem!
- However, triumph of QCD: discovery that  $\alpha_s$  is NOT constant, but depends on the separation of the interacting particles  $\Rightarrow$  ‘running’ coupling constant:
  - $\alpha_s$  is large at large distances (larger than proton)  $\Rightarrow$  ‘confinement’
  - $\alpha_s$  is small at very short distances (smaller than proton)  $\Rightarrow$  (‘asymptotic freedom’)

- Complete QCD Lagrangian

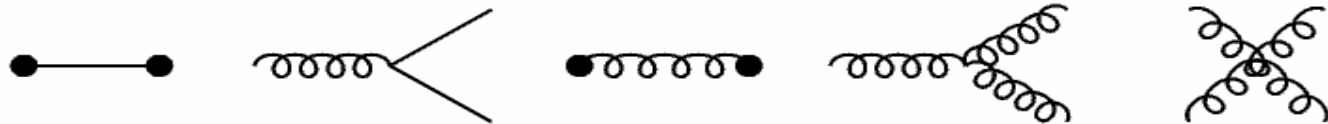
$$\mathcal{L} = [i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi] - \left[ \frac{1}{4} \frac{1}{6\pi} F^{\mu\nu} F_{\mu\nu} \right] - (q \bar{\psi} \gamma^\mu \bar{\lambda} \psi) A_\mu$$

with

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - \frac{2q}{\hbar c} (\vec{A}^\mu \times \vec{A}^\nu)$$

- 1<sup>st</sup> part is analog to photon field in QED; 2<sup>nd</sup> term is new: quadratic in gluon field

$$\mathcal{L}_{QCD} = (q\bar{q}) + g(q\bar{q}F) + (F^2) + g(F^3) + g^2(F^4)$$

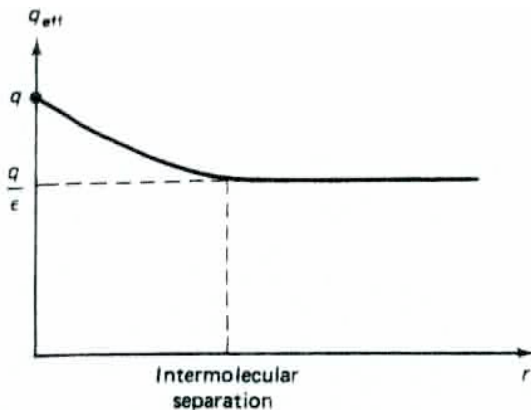
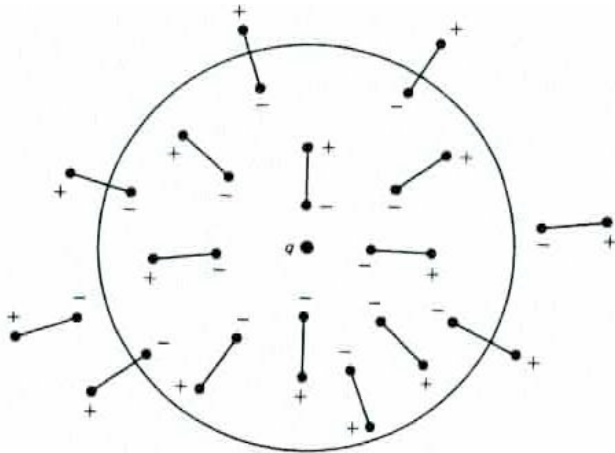


- quarks
- quark-gluon
- gluons
- gluon-gluon self interaction

- Lagrangian describes three equal-mass Dirac fields (the three colors of a given quark flavor) with eight massless vector fields (the gluons)
- Lagrangian applies to one specific quark flavor; need altogether six replicas of  $\psi$  for the six quark flavors

# Detour to QED

- Also in electrodynamics: effective coupling also depends on distance
  - Charge  $q$  embedded in dielectric medium  $\epsilon$  (polarizable)



- medium becomes polarized
- Particle  $q$  acquires halo of negative particles, partially screening the charge  $q$
- at large distance charge is reduced to  $q / \epsilon$
- in QED: vacuum behaves like dielectric
- full of virtual positron-electron pairs
- virtual electron attracted to  $q$ , positron repelled



- This vacuum polarization screens partially the charge at distances larger than  $h/mc = 2.4 \cdot 10^{-10}$  cm (Compton wavelength of electron)
- Measurable, e.g. in structure of hydrogen levels
- NOTE: we measure the 'screened' charge, not the 'bare' charge

# Coupling constants: QED vs QCD

- QED : coupling constants modified by virtual effects ('loop diagrams')



which 'screens' the electric charge and modifies the coupling constant as a function of the distance ( or equivalently; of the momentum transfer of a reaction); observable: Lamb shift; anomalous magnetic moment

$$\alpha_{QED} = \frac{\alpha(0)}{1 - [\alpha(0) / 3\pi] \ln[|q^2| / (mc)^2]} \quad \text{for } |q^2| \gg (mc)^2$$

- Coupling constant  $\alpha_{QED}$  varies only very weakly with  $q^2$

- Diagrams analogous to QED contribute to vacuum polarization
- $qqg$  vertex: contributes to increasing coupling strength at short distance
- In addition: direct  $gg$  vertex



- Competition between quark polarization diagrams,  $\alpha_s \uparrow$  and gluon polarization,  $\alpha_s \downarrow$  at short distances



# QCD vacuum polarization and 'Camouflage'

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- In polarized medium quark continuously emits and reabsorbs gluons, changing constantly its color
- Color-charged gluons propagate to appreciable distances, spreading the color charge of the quark, camouflaging the quark, which is source of the color charge
  - The smaller the region around the quark the smaller the effective color charge of the quark → color charge felt by quark of another color charge approaching the quark will diminish as the quark approaches the first one
- Net effect: competition between screening and camouflage
- QCD: critical parameter  $a = 2 f$  (of flavors) –  $11 n$  (number of colors)
  - if  $a$  is positive (as in QED), coupling increases at short distance
  - in SM:  $n = 3$ ,  $f = 6$ ;  $a = -21$ ; QCD coupling decreases at short distance

# Coupling constants: QCD

- The screening effects happen in QCD (quark-antiquark loops), BUT in addition due to gluon couplings



- with the result for the coupling constant

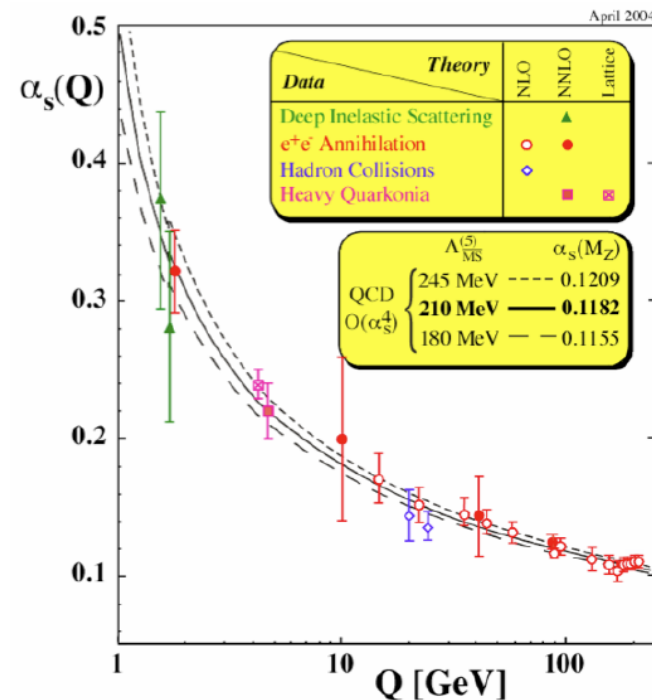
$$\alpha_{QCD} = \alpha_s = \frac{\alpha_s(\mu^2)}{1 + \left[ \alpha(\mu^2) / 12\pi \right] (11n - 2f) \ln \left[ |q^2| / \mu^2 \right]} \quad \text{for } |q^2| \gg \mu^2$$

- With  $n =$  colors (3 in SM) and  $f =$  number of quark flavors (6 in SM);  $\mu$  is a reference value around which  $\alpha_s$  is evaluated.
- At large  $q^2$   $\alpha_s$  becomes less than 1  $\rightarrow$  perturbation theory is applicable;  $\rightarrow$  **Asymptotic Freedom**.
- There are equivalent Feynman rules for quantitative calculations: perturbative QCD is quantitatively tested at the  $<1\%$  level

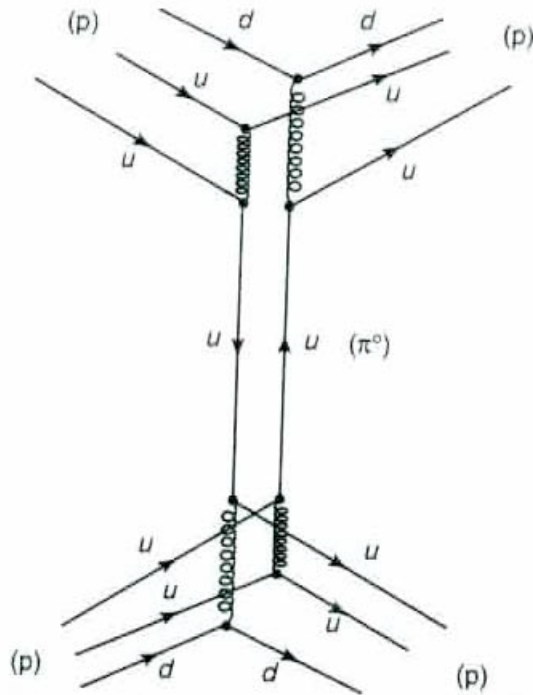
# Coupling constants: QCD

- At small  $q^2$   $\alpha_s$  becomes very large -> **confinement**
- The exact proof of confinement is very difficult to demonstrate within the frame work of QCD and is a very active topic of current research in strong interactions

World Average :  $\alpha_s(M_Z) = 0.1183 \pm 0.0027$



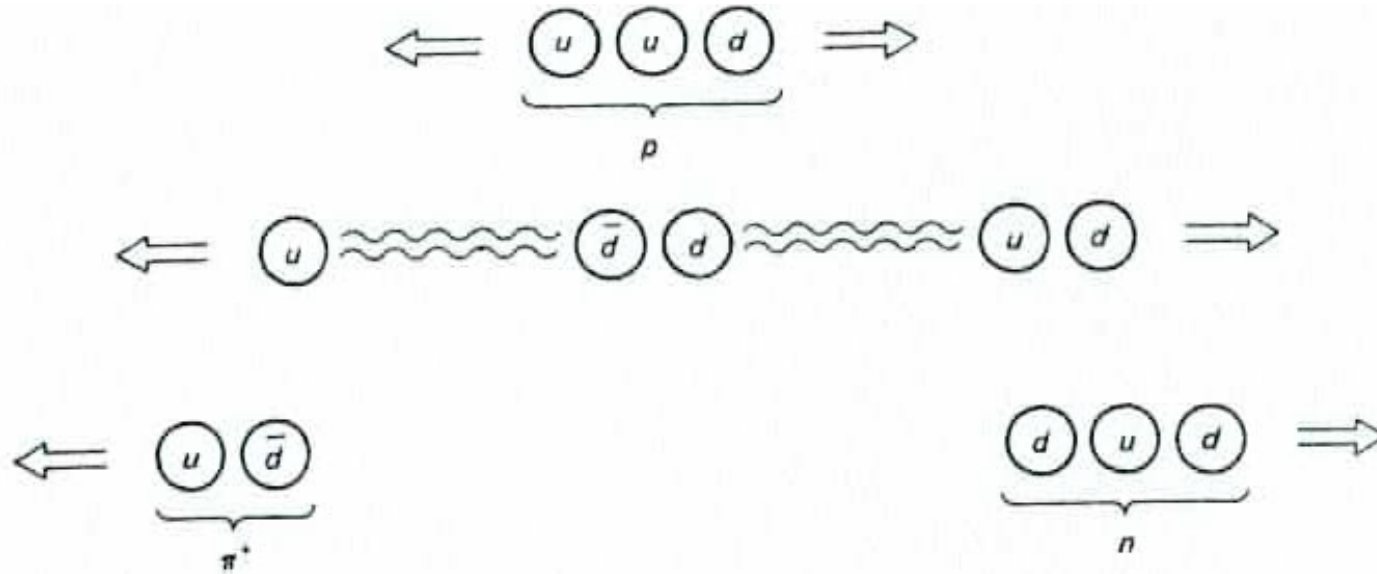
# QCD: One more difference



- quarks are confined in colorless packages
- experimental observations are indirect and are complicated manifestations of QCD
- force between two protons involves diagrams of the type shown
- reminiscent of the Yukawa  $\pi$ -exchange model

- QCD: theory must *prove* confinement: ongoing major task of theoretical research!
- QCD prediction at very high temperature (short range) phase transition to deconfined 'Quark-Gluon Plasma' -> subject of intense theoretical and experimental current research

# Possible Scenario for Quark Confinement



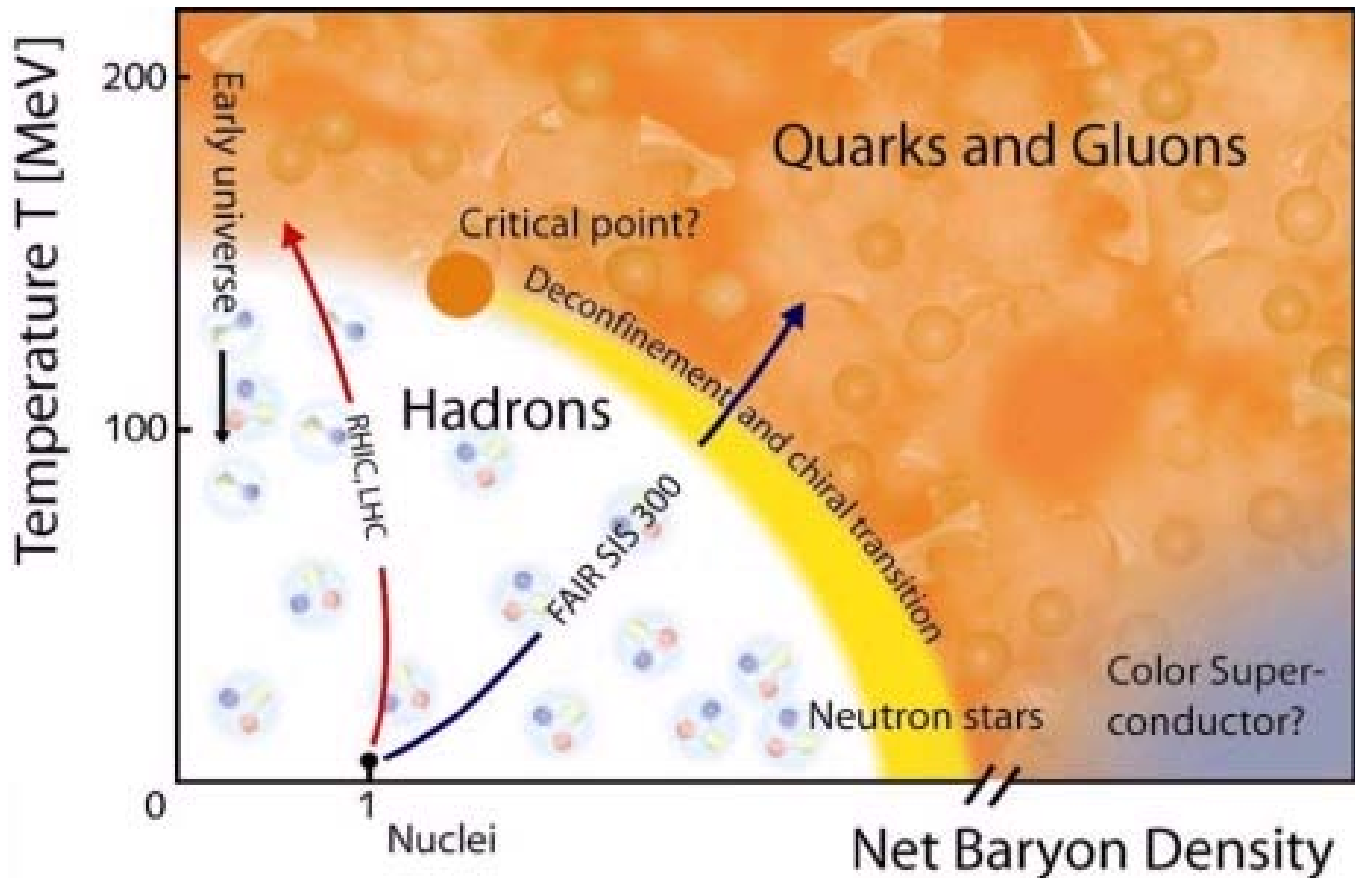
- Concept for *proof* of quark confinement: potential energy increases without limit as quarks are pulled farther and farther apart  $\rightarrow$  energetically more favorable to produce quark-antiquark pairs
- Conclusive proof for confinement still lacking: long-range interaction difficult to treat theoretically

# Quark-Gluon Plasma

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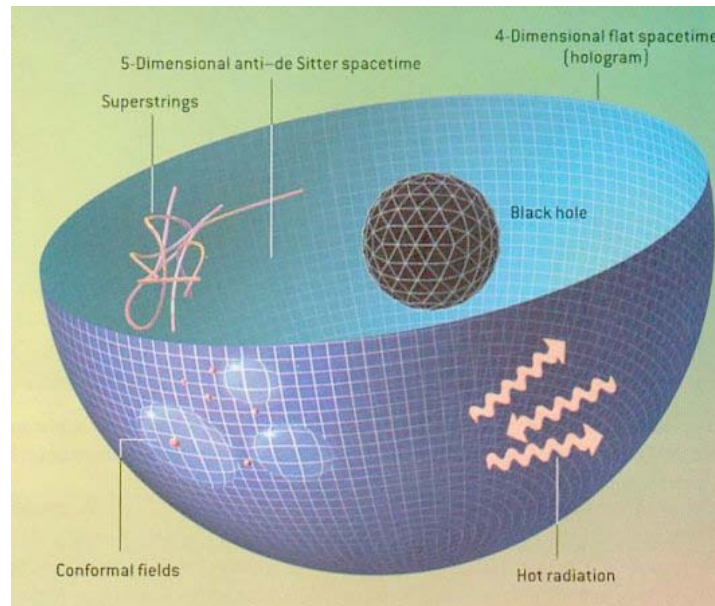
- One 'golden' prediction of QCD is the 'Quark-Gluon Plasma', deconfined quarks and gluons at very high density or temperature
  - $T \sim 170 \text{ MeV} \sim 10^{12} \text{ K}$
  - thought to have been the primordial matter during the first microsecond after the Big Bang
- Considered to be created in very energetic collisions of heavy nuclei (e.g. lead ions)
  - Was an active program at the CERN SPS; now actively being pursued at RHIC (Relativistic Heavy Ion Collider) at Brookhaven, USA
  - Major research activity at the LHC with one dedicated facility

Phase Diagram of Quark Gluon Plasma



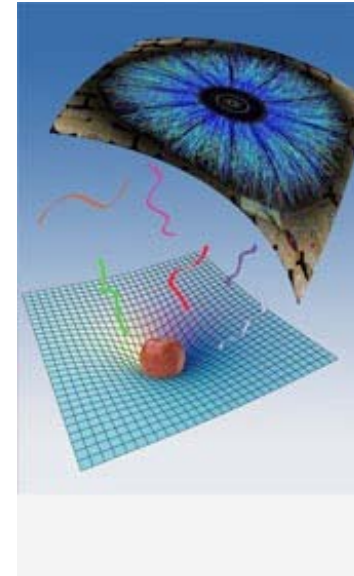
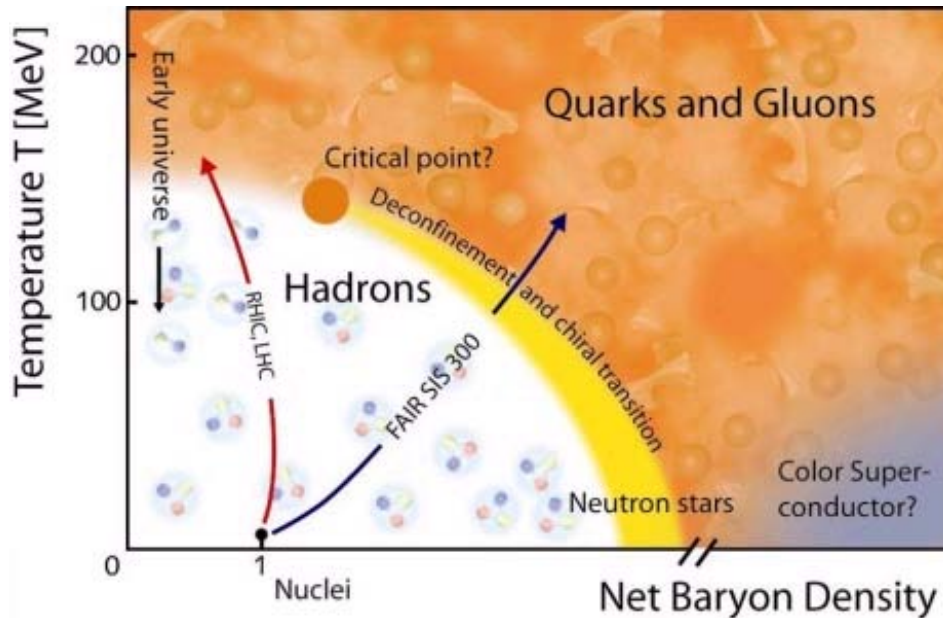
At sufficiently high temperature Nuclear Matter undergoes a phase transition to deconfined quarks and gluons: Quark-Gluon Plasma;

- Most-cited theoretical development of last decade  
Correspondence between anti-deSitter Gravity and conformal Quantum Field-Theories: AdS/CFT
  - Discovery within frame of Superstring-Theory
  - Strongly interacting Quantum Fieldtheorie (z.B. QCD) in 3 space and 1 time dimension →  
equivalently described with 5-dimensional Gravity theory





# Applying the AdS/CFT correspondence: Black Holes $\leftrightarrow$ Quark-Gluon Plasma



- Spectacular application: strongly coupled Quark-Gluon Plasma is described with the physics of black holes in 5 dimensionen (and vice-versa)
- Successful prediction: viscosity of Quark-Gluon Plasma
- Quark-Gluon Plasma is a very active field of study at LHC