Symmetries and Conservation Laws

Symmetries and Conservation laws

Groups

Angular Momentum and extensions

Discrete Symmetries

Parity, Charge Conjugation, Time Reversal

Violations of Symmetries

The CTP Theorem

Symmetries

- Symmetry: if a set of transformations, when applied to a system, leaves system unchanged → transformation is a symmetry of system
- Symmetries play an important role in particle physics, partly because they are related to conservation laws
- Understanding the origin of conservation laws guides the formulation of the quantitative description of the particle interactions: the inverse is also true: from symmetries of the interaction-> conservation laws
- Symmetry of crystals: shape is a 'static' symmetry
- Dynamical symmetries: associated with motion, interaction
- Newton: spherical symmetry of gravitational law NOT exhibited in motion of planets (orbits are elliptical !), but in the set of all possible motions ⇒ in the equations of motion ⇒
  - underlying symmetry only indirectly exhibited

Example of a symmetry: Translation Invariance

- Lagrangian of system with *n* degrees of freedom
  - n-coordinates; n-velocities
- $L = L(q_i, \dot{q}_i)\ i = 1,2,\ldots n$
- Associated momenta, or momenta conjugate to the coordinates q_i

$$p_i = \partial L / \partial \dot{q}_i \qquad i = 1,\ldots n$$

- Dynamical equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \Rightarrow \frac{dp_i}{dt} = \frac{\partial L}{\partial q_i}$$

- If Lagrangian of this system is independent of a particular coordinate q_m

$$\frac{\partial L}{\partial q_m} = 0 \Rightarrow \frac{dp_m}{dt} = 0$$

- Independent of a particular coordinate ⇒ translation invariant ⇒ conjugate momentum conserved

Connection between Symmetries and Conservation Laws: Noether’s Theorem

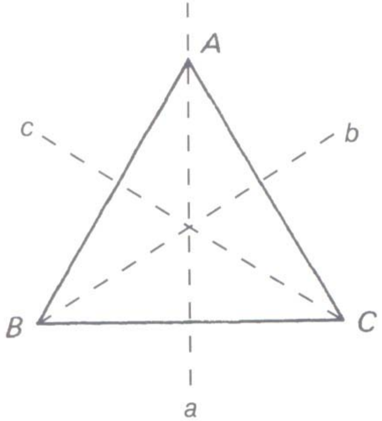
- Every symmetry of nature yields a Conservation Law
- Converse is also true: every conservation law reflects an underlying symmetry

Symmetry		**Conservation Law**
Translation in time	$\longleftrightarrow$	Energy
Translation in space	$\longleftrightarrow$	Momentum
Rotation	$\longleftrightarrow$	Angular Momentum
Gauge Transformation (Electrodynamics)	$\longleftrightarrow$	Charge

Symmetry: Definition

- Symmetry: is an operation, which – performed on a system – leaves system invariant, i.e. carries it into a configuration, which is indistinguishable from original one

- Example: operation on equilateral triangle



- unchanged under clockwise rotation of 120° (R^+)
- unchanged under counter clockwise rotation (R^-)
- unchanged under flip about vertical axis *a* (R_a)
- unchanged under flip about vertical axis *b* (R_b)
- unchanged under NO operation: identity (*I*)
- unchanged under combined operation
- Clockwise rotation under 240^0 ($R^+ R^+$) = R^- …all possible symmetry operations defined by above operation

Set of All Symmetry Operations ⇒ GROUP

- Set of all symmetry operations has following properties
  - closure: if R_i and R_j are in the set ⇒
    product $R_i\, R_j$ (first perform R_j , then R_i) is also in the set
    $R_i \; R_j = R_k$ *(closure is in German: ‘ Geschlossenheit’)*
  - identity: element I exists, such that $I\, R_i = R_i\, I = R_i$ for all R_i
  - inverse: for every element R_i → inverse, R_i^{-1}, exists, such that
    $$R_i \; R_i^{-1} = R_i^{-1}\, R_i = I$$
  - associativity: $R_i\,(\, R_j\, R_k) = (\, R_i\, R_j\,)\, R_k$
- Rules are defining properties of a mathematical group G
  - Mapping of $G = \{R_i\, ,\, R_j\, ,\, R_{k\, ,...}\}$ onto the group of linear transformations in Vectorspace ⇒ R_i -> $D\,(R_i)$… D are frequently matrices

Group Theory: Classification of symmetries

- Abelian Group: group elements commute: $R_i \; R_j = R_j \, R_i$
  - translation in space and time ⇒ Abelian Group

- Non-Abelian Group: $R_i \, , R_j \neq R_j \, R_i$
  - rotations in three dimensions do not commute

- Finite Groups: example ‘Triangle’: has six elements

- Continuous Groups: e.g. rotations in a plane

- Discrete Groups: elements labelled by index that takes only integer values

Most Groups in Physics ⇒ represented by Groups of Matrices

- In General: every group *G* can be represented by a group of matrices: for every group element *a* ⇒ matrix M_a
- Lorentz Group: set of 4 x 4 Λ matrices $x^{\mu'} = \Lambda^{\mu}_{\nu} x^{\mu}$
  - transformation in 4-dimensional space
- Unitary Groups U(n): collection of all unitary n x n matrices
  - unitary matrix: inverse $U^{-1} \equiv$ transpose conjugate $\tilde{U}*$
- Special Unitary Groups SU(n): unitary matrices with determinant 1
  - Gell-Mann’s eightfold way corresponds to representations of SU(3)
- Real Unitary Groups O(n): orthogonal matrices: $O^{-1} = \tilde{O}$
- Real Orthogonal Groups SO(n): determinant 1
  - SO(3): rotational symmetry of our world, related through Noether’s theorem to conservation of angular momentum,

Example 1: SO(3)

- Rotation in 3-dimensional space can be described with orthogonal, unimodular 3 x 3 matrices R

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad RR^T = 1$$

- Consider rotation about z-axis

$$R_Z = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- R_Z can also be presented as $R_Z(\theta) = e^{i\theta J_Z}$
- J_Z are 'Generators' of the group R_Z

with $J_Z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, similarly for $J_X,\ J_Y$

Example : SU(2)

- SU(2): complex, unitary, unimodular 2 x 2 matrices
- $U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with *a, b, c, d* …. complex
- In general: 8 parameters; however
  unimodular: det U = 1; unitary: $U^{+}U = 1$
- $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$ with $|a|^2 + |b|^2 = 1$; 3 free parameters
- $U = e^{i\frac{\theta_i}{2}\sigma_i}$ ⇒ Pauli Matrices, transformation in spinor space

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin $\frac{1}{2}$

Spin $\frac{1}{2}$: most important spin system: proton, neutron, electron, quarks

- particle with $s = \frac{1}{2}$: $m_S = \frac{1}{2}$ ('spin up'), $m_S = -\frac{1}{2}$ ('spin down')
- states can be presented by arrow: ↑ ; ↓
- better notation: two-component column vector, or spinor

$$\left|\frac{1}{2}\,\frac{1}{2}\right\rangle = \begin{pmatrix}1\\0\end{pmatrix};\ \left|\frac{1}{2}\,-\frac{1}{2}\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- most general case of spin $\frac{1}{2}$ particle is linear combination

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix};\quad \alpha, \beta \text{ complex numbers}$$

- measurement of s_Z can only return value of $+\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$
- $|\alpha|^2$ is probability that measurement of s_Z yields $+\frac{1}{2}\hbar$

  $|\beta|^2$ is probability that measurement of s_Z yields $-\frac{1}{2}\hbar$

  therefore: $|\alpha|^2 + |\beta|^2 = 1$

Flavour Symmetries

- Heisenberg, 1932: observed that proton and neutron (apart from charge) are almost identical
  - M_p = 938.28 MeV/c^2; M_n = 939.57 MeV/c^2
  - proposed to regard proton and neutron as two 'states' of the same particle, the 'nucleon'
  - nucleon written as a two-component spinor

$$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

  - by analogy to spin ⇒ 'Isospin'
  - Nucleon carries isospin ½
  - I is vector not in ordinary space, but in abstract 'isospin space', with I_1 , I_2 , I_3
  - Machinery developed for Angular Momenta can be applied

Concept of Isospin

- Heisenberg's proposition: strong interactions are invariant under solution in isospin space (analog to: electrical forces are invariant under rotation in ordinary configuration space)

- Isospin invariance is 'Internal Symmetry' of system

- Noether's theorem

  strong interactions is invariant under rotation in isospin space ⇒ isospin is conserved in all *strong interactions*

- Isospin assignment follows isospin assignment of the quarks
  - *u* and *d* form doublet: $u = \left| \frac{1}{2} \frac{1}{2} \right\rangle;\ d = \left| \frac{1}{2} - \frac{1}{2} \right\rangle$
  - all other quarks carry isospin zero

- Isospin formalism associated with SU(2)
  - strong interactions are invariant under internal symmetry group SU(2)

Dynamical Consequences of Isospin

- Two-nucleon system
  - symmetric isotriplet: $|11\rangle = pp; |10\rangle = \left(\frac{1}{\sqrt{2}}\right)(pn+np); |1-1\rangle = nn$
  - antisymmetric isosinglet: $|00\rangle = \left(\frac{1}{\sqrt{2}}\right)(pn-np)$ is the deuteron (if it were in triplet, all three states would have to occur)
  - Isosinglet state is totally insensitive to rotation in isospin space
- Nucleon-nucleon scattering
  - $p+p \to d+\pi^+; p+n \to d+\pi^0; n+n \to d+\pi^-$
  - deuteron *I* = 0; isospin states on right side are $|11\rangle, |10\rangle, |1-1\rangle$, on left side $pp = |11\rangle; nn = |1-1\rangle, pn = \left(\frac{1}{\sqrt{2}}\right)(|10\rangle + |00\rangle)$
  - only *I* = 1 combination contributes (final state is pure *I* = 1)
    - o scattering amplitude in ratio $1:\frac{1}{\sqrt{2}}:1$ and cross sections (square of amplitudes) $1:\frac{1}{2}:1$, consistent with measurement

Dynamical Consequences of Isospin: Δ(1232)

- Four members of the Δ(1232) family, mass 1232 +-1 MeV/c^2 , corresponding to four charged states, four I_3 projections of = I=3/2 multiplet

- If isospin is good symmetry ⇒ total decay rates must be identical

Charge State of Δ	I_3	Final State	Expected Rate	Solution
Δ^{++} (uuu)	$\frac{3}{2}$	$p\pi^+$	1	1
Δ^{+} (uud)	$\frac{1}{2}$	$p\pi^0$	x	$\frac{2}{3}$
		$n\pi^+$	$1-x$	$\frac{1}{3}$
Δ^{0} (udd)	$-\frac{1}{2}$	$p\pi^-$	y	$\frac{1}{3}$
		$n\pi^0$	$1-y$	$\frac{2}{3}$
Δ^{-} (ddd)	$-\frac{3}{2}$	$n\pi^-$	1	1

- Ratio for decays involving a p or n in final state are the same

$$1 + x + y = 1 - x + 1 - y + 1$$

- Ratio for decays involving π^+, π^0 or π^- are the same

$$1 + 1 - x = x + 1 - y = y + 1$$

- Unique set of consistent solution shown in table, in agreement with measurements

Extension of Isospin Concept

- Eight baryons have approximately equal mass
  - tempting to regard them as a supermultiplet, belonging to the same representation of an enlarged symmetry group, in which SU(2) of isospin would be a subgroup

- Gell-Mann found that the corresponding symmetry group is SU(3)
  - octets are represented by the 8-dimensional representation of SU(3), i.e. the baryons belonging to this octet are transformed by the 8-dimensional representation of SU(3)
  - decuplets are represented by the 10-dimensional representation of SU(3)

Extension of Isospin Concept

- Difficulty: no particles fall into the fundamental, 3-dimensional representation (in contrast to nucleon) ⇒ emerging idea of quarks

- Gell-Mann: quarks: *u, d, s* transformed according to the 3-dimensional representation of SU(3) which breaks down into isodoublet (u,d) and isosinglet (s) under SU(2)

- In this concept baryons, consisting of three quarks, $qqq...R(3)\otimes R(3)\otimes R(3) = 3\otimes 3\otimes 3 = 1+8+8+10$ decomposed into irreducible representations, with which the octets and decuplets can be identified

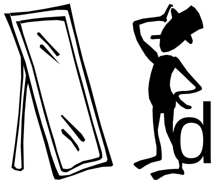
Caveat in this Hierarchy

- Isospin, SU(2), symmetry is very good symmetry; mass of members of isospin multiplets differ by only few %

- Discrepancies become very large, when including the heavier quarks in this concept ⇒ can be traced to the bare quark masses

Quark Flavour	Base Mass	Effective Mass
u	2	336
d	5	340
s	95	486
c	1300	1550
b	4200	4730
t	174000	177000

- Effective mass is consequence of strong interactions due to confinement inside hadrons
  - effective mass of *u, d, s,* almost equal
  - Effective mass of *c, b, t,* very different
- We have no fundamental explanation for the value of the bare quark masses

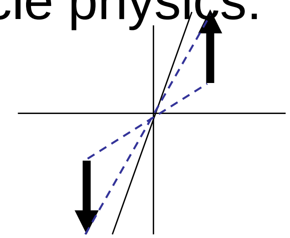


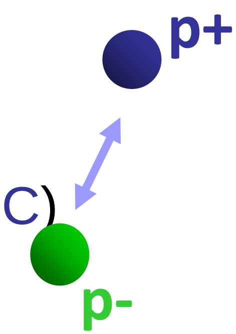
discrete symmetries

Discrete symmetry: symmetry that describes non-continuous changes

Fundamental symmetry operations in particle physics:

- parity transformation (spatial inversion P)





- particle-antiparticle conjugation (charge conjugation C)

- time inversion (T)

According to the kind of interaction, the result of such a transformation may describe a physical state occurring with the same probability (“the symmetry is conserved”) or not (“the symmetry is broken” or “violated”).

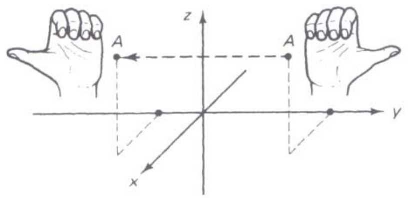
M. Jeitler

Discrete Symmetries: Parity

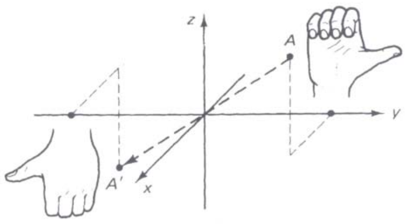
- Transformation on state may describe a physical state occurring with
  - the same probability -> ‘symmetry is conserved’
  - or not: ‘symmetry is not conserved’, ‘is broken’, ‘is violated’
- Prior to 1956: ‘obvious’ that laws of physics are ambidextrous: the mirror image of a physical process is also a perfectly possible process
- Mirror symmetry (‘Parity Invariance’) considered to be ‘self-evident’.
- Lee and Yang: what are the experimental proofs of this assumption ?
  - ample evidence for parity invariance in electromagnetic and strong processes
  - NO evidence in weak interactions
  - proposed experiment to settle the question -> result: Parity is violated in weak interaction processes!
- Nobel Prize for Lee and Yang in 1957

Parity

- ‘Reflection’: arbitrary choice of mirror plane → better to consider
- Inversion = reflection, followed by 180° rotation
- P … parity operator, denoting inversion
  - applied to vector *a* *P(a)* = -*a* (‘polar’ vector) (vector in opposite direction
  - applied to cross product *c* = *a* x *b* *P(c)* = *c* (‘axial’ vector) (magnetic field; angular momentum are axial vectors)



(a) Reflection (in the x-z plane)
(x, y, z) → (x, −y, z)



(b) Inversion (x, y, z) → (−x, −y, −z)

inversion: every point is carried through origin to diametrically opposite location

Parity

- *P* (**a**•**b**) = (**a**•**b**) scalar
- *P* (**a**•[**b** x **c**]) = - **a**[**b** x **c**] pseudoscalar
- $P^2 = I$; parity group has two elements: *I, P*
  - eigenvalues of *P* are +1, -1

scalar:	$P(s) = s$	$P = +1$
Pseudoscalar:	$P(p) = -p$	$P = -1$
Vector (polar vector):	$P(\mathbf{v}) = -\mathbf{v}$	$P = -1$
Pseudovector (axial vector):	$P(\mathbf{a}) = \mathbf{a}$	$P = +1$
Angular momentum	$\vec{L} = \vec{r} \times \vec{p} \rightarrow P \rightarrow -\vec{r} \times -\vec{p} = \vec{L}$	

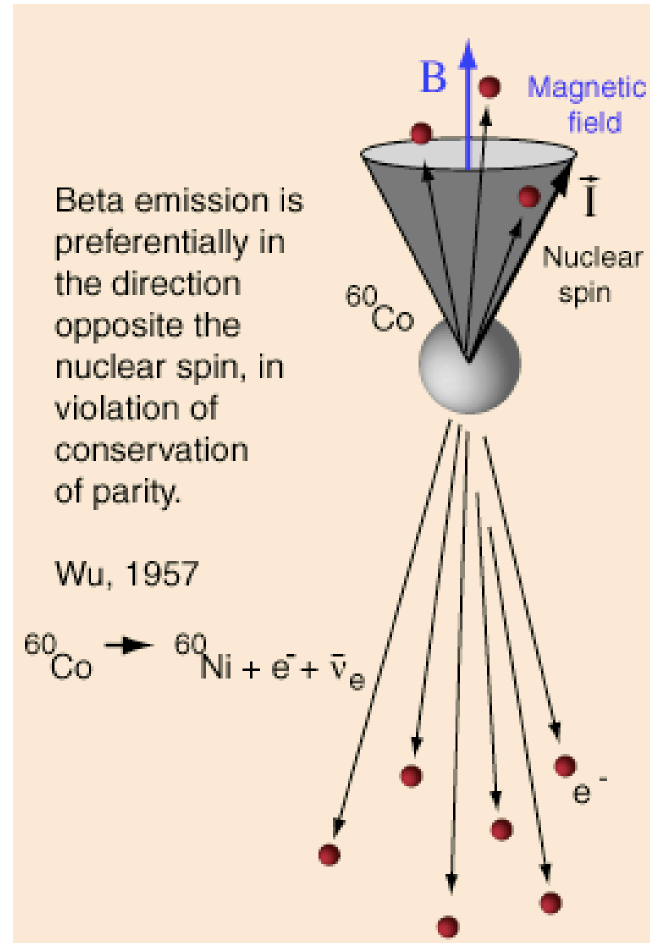
- Hadrons are eigenstates of *P* and can be classified according to their eigenvalue
- *P* (fermion) = -*P* (antifermion) *P* (quark) = +1, *P* (antiquark) = -1
- *P* (photon) = -1 (S=1; is vector particle, represented by vector potential)
- *P* (composite system in ground state) = *P* (of product)
- *P* is multiplicative

Observation of Parity Violation by Wu and Collaborators (1957)

- Study decay of $^{60}Co \rightarrow {}^{60}N_i + e^- + \overline{\nu}_e$
- polarized matter
  - ^{60}Co at 0.01 Kelvin inside solenoid
    - high proportion of nuclei aligned
- ^{60}Co (J=5) → ^{60}Ni* (J=4) (similar to β-decay
  - electron spin **σ** points in direction of ^{60}Co spin **J**
  - conservation of angular momentum
  - degree of ^{60}Co alignment determined from observation of ^{60}Ni* γ-rays
- observed electron intensity:

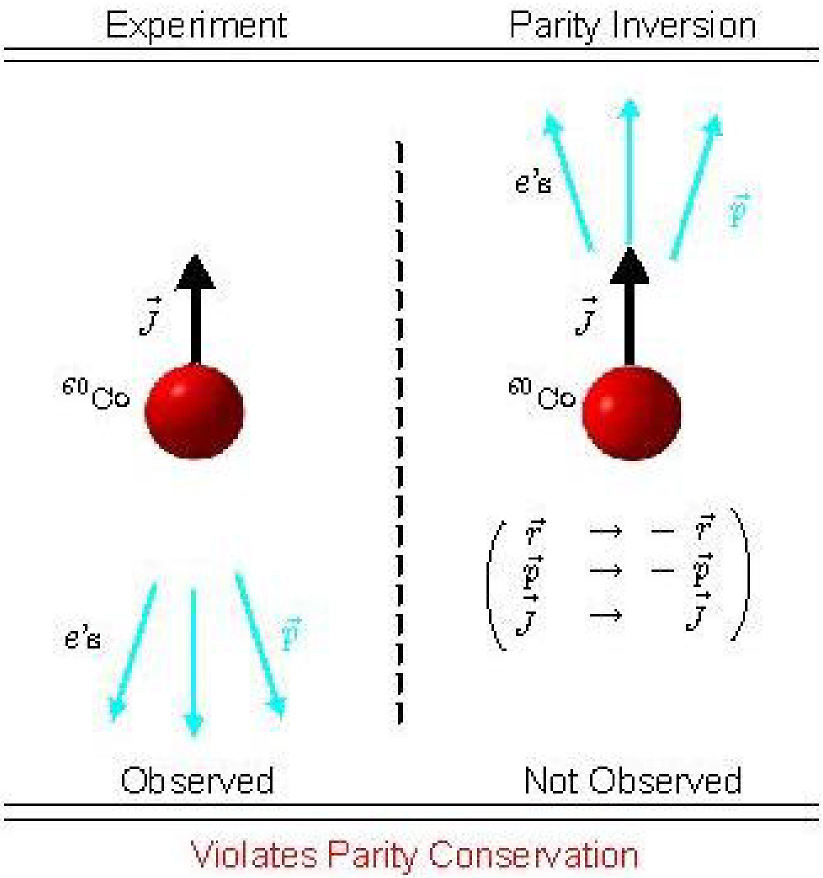
$$I(\vartheta) = 1 - \left(\frac{\sigma . \mathbf{p}}{E} \right) = 1 - \frac{v}{c} \cos \vartheta$$

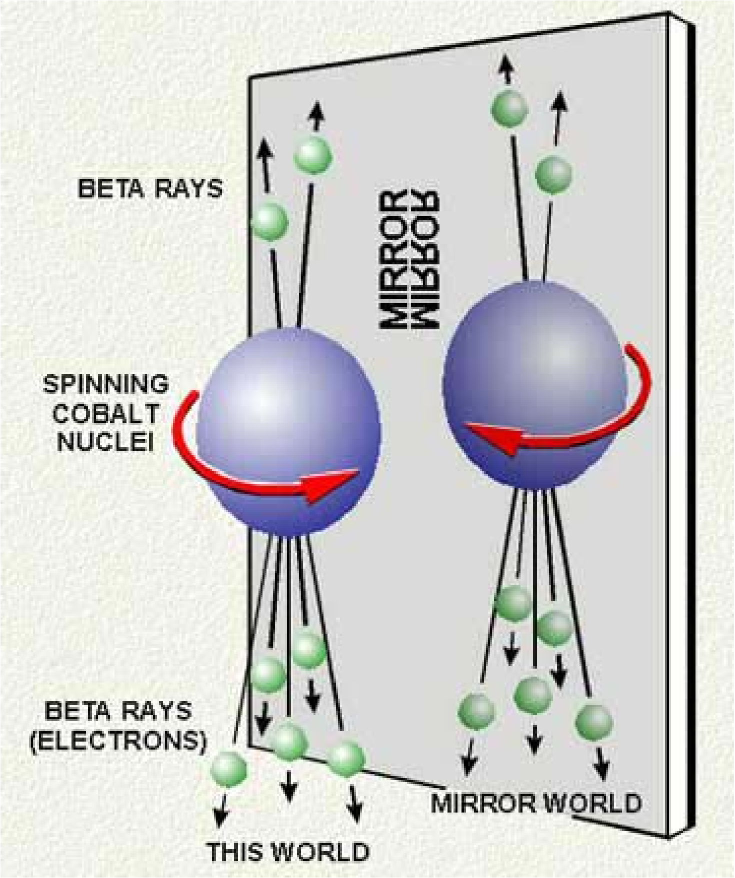
  - ϑ: angle between electron (**p**) and spin (**J)**
- If Parity were conserved, would expect electrons equally frequently emitted in both directions
- Under Parity, **p** changes sign, but not Spin → expectation value of angular distribution changes sign – if right- and left-handed coordinate system are equivalent → only possible, if distribution is uniform → Observed distribution not uniform → Parity is violated



M. Jeitler

Parity violation





Wu-Ambler et al Experiment

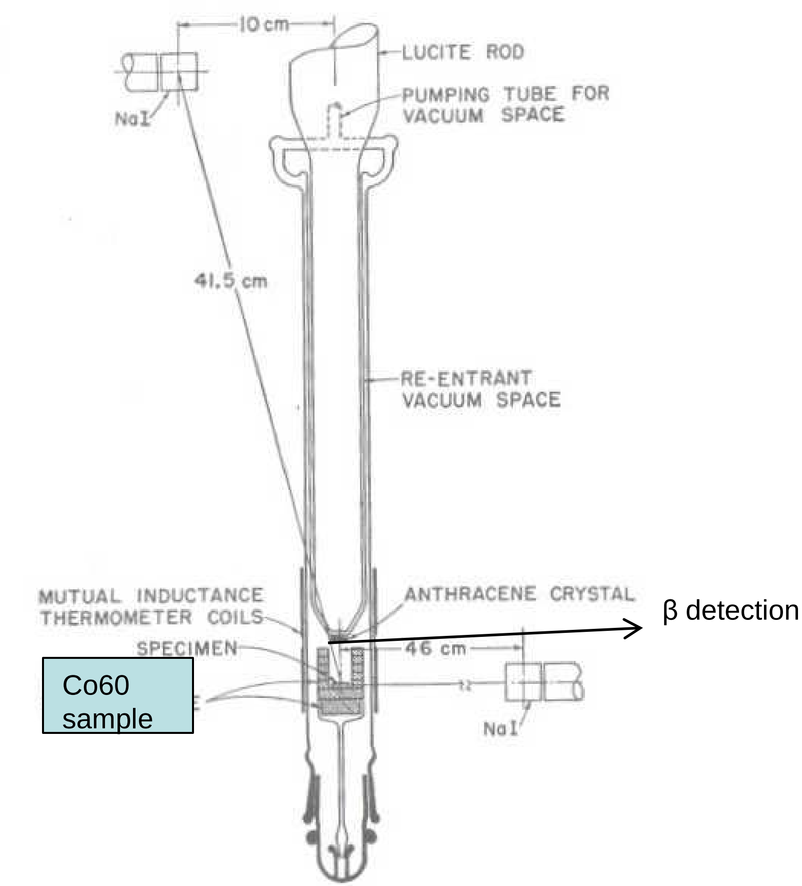


FIG. 1. Schematic drawing of the lower part of the cryostat.

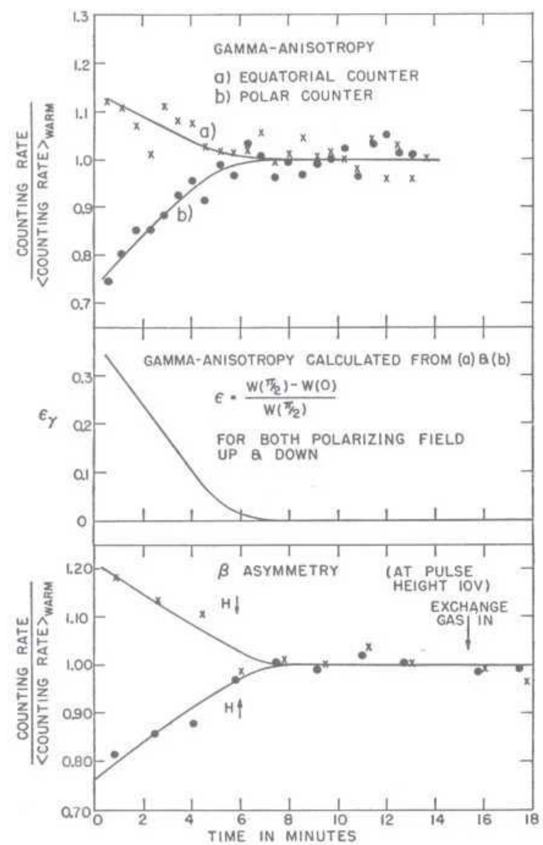
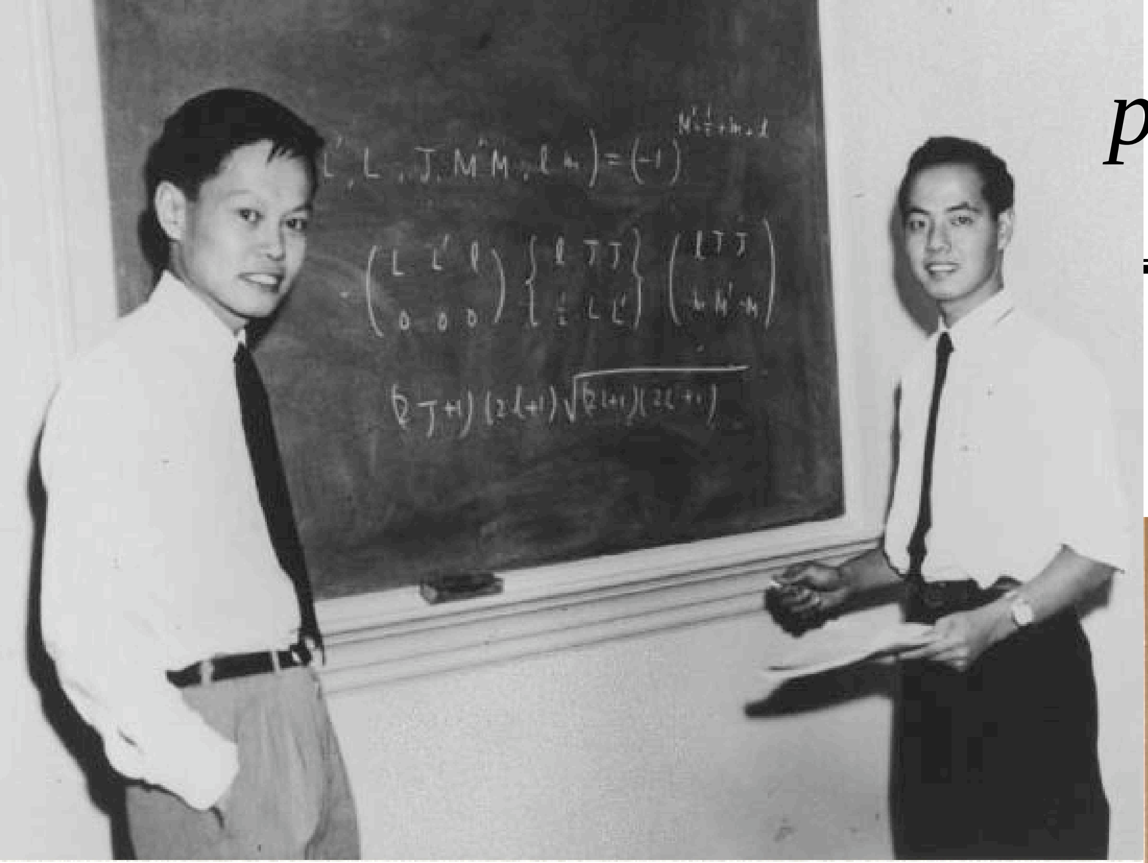


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

Detail of apparatus

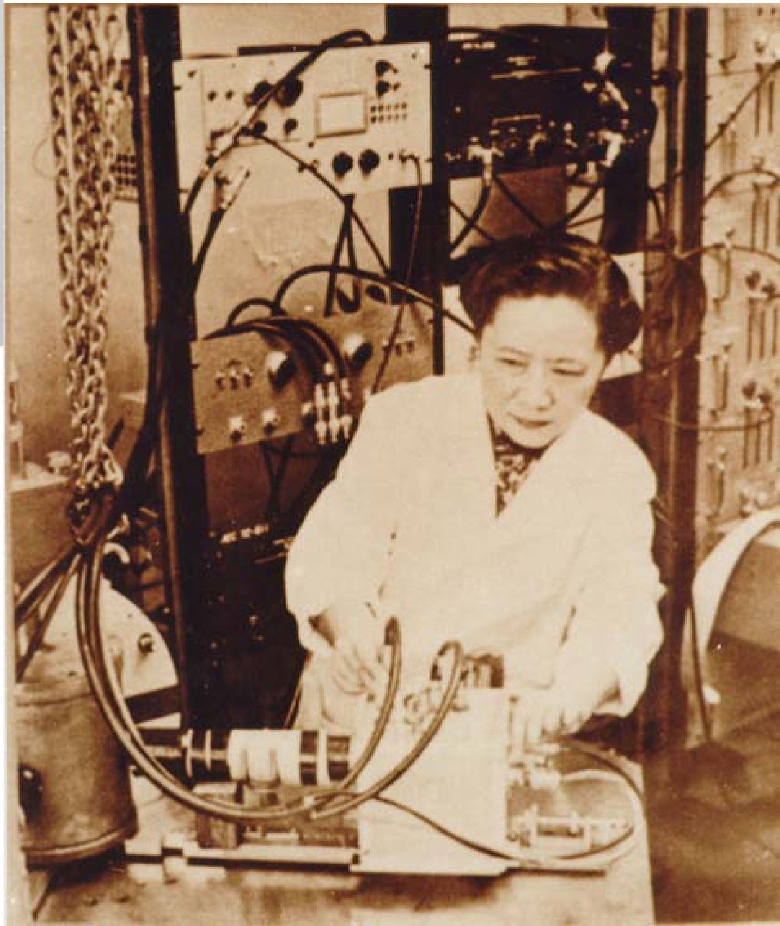
Published results

parity violation



Chen Ning **Yang** and Tsung-Dao **Lee**
(Nobel prize 1957)

Chien-Shiung **Wu**



Parity Violation in Weak Interaction

- Parity violation is feature of weak interaction
  - is 'maximally' violated
  - most dramatically revealed in the behaviour of neutrinos

- 'Helicity' of neutrinos
  - particles with spin s travelling with velocity along z-axis
  - value of m_s/s = helicity of particle
  - particle with spin ½ can have
    - helicity 1 ($m_s = ½$) ('right-handed')
    - helicity -1 ($m_s = -½$) ('left-handed')

(a) Right-handed (b) Left-handed

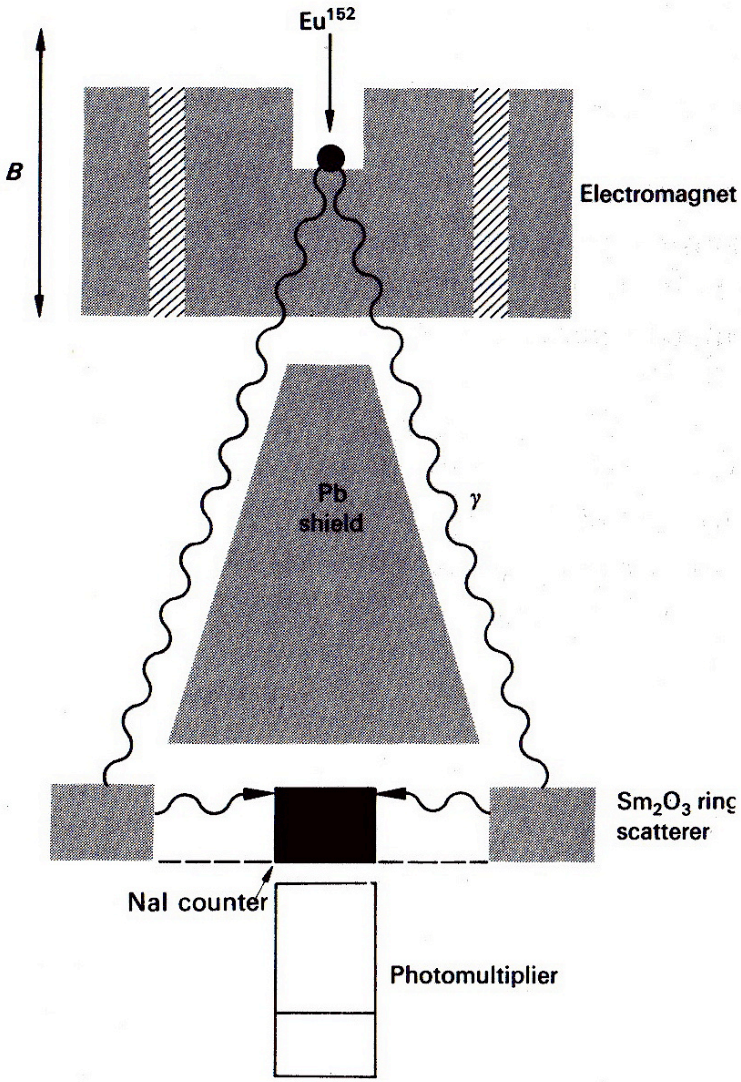
- Helicity of massive particle is NOT Lorentz-invariant

- Helicity of massless particle, travelling with v=c, is Lorentz-invariant

Helicity of Neutrinos

- Photons can have helicity + or -, representing left and right circular polarization
- Neutrinos are found to be always left-handed (helicity H= -1)
  Antineutrinos are found to be always right-handed (helicity H= 1)
- Parity violation in weak interaction is a consequence of this fact
  - Mirror image of neutrino does not exist
- Observation of neutrino helicity : $\pi^- \to \mu^- + \overline{\nu}_\mu$
  - pion at rest $\Rightarrow \mu$ and $\overline{\nu}$ energy back-to-back, spin of pion s=0 : spin of muon and antineutrino opposite aligned
  - if muon is observed to be right-handed $\Rightarrow$ $\overline{\nu}$ must be right-handed

Helicity of Neutrino: a marvellous landmark experiment



Experiment carried out by Goldhaber et al in 1958

^{152}Eu + (K-capture) e^- → $^{152}Sm^*$ + ν_e

$^{152}Sm^*$ emits) 0.96 MeV γ →

1) measurement of direction of γ → measurement of direction of neutrino (back-to-back)

2) measurement of helicity of γ determines helicity of neutrino

3) helicity of γ : Compton scattering in iron (below the ^{152}Eu source) depends on helicity of γ relative to spins of iron; scattering changes γ energy →
changing the magnetic field changes spins of iron → changes Compton scattering → measured via resonant absorption in Sm_2O_3 -ring determines helicity of γ →
helicity of neutrino H (ν_e) = - 1.0 +- 0.3

Helicity of Neutrino: a marvellous landmark experiment

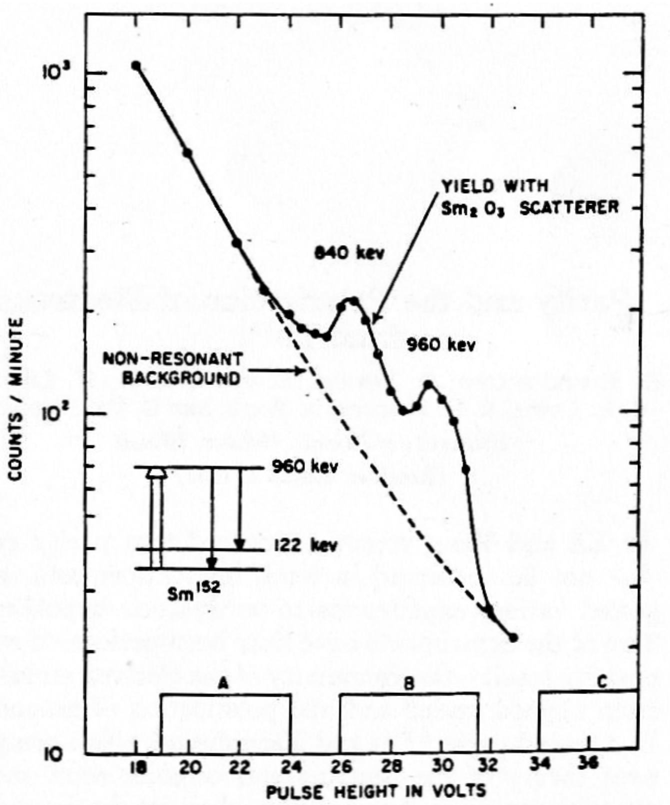


FIG. 2. Resonant-scattered γ rays of Eu^{152m}. Upper curve is taken with arrangement shown in Fig. 1 with unmagnetized iron. Lower curve shows nonresonant background (including natural background).

Count rate in $Sm_2 O_3$ – analyzer as function of the polarisation of B-field → determines helicity of gamma → helicity of neutrino

Charge Conjugation (Particle –> Antiparticle)

- Classical electrodynamics
  - invariant under charge in sign of all electric charges
  - potential, fields reverse sign
  - forces are invariant (charge factor in Lorentz law)
- Elementary particle physics: generalization of ‘changing sign of charge’
  - Charge conjugation *C* converts particle into antiparticle $C|p\rangle = |\bar{p}\rangle$
- Note: charge conjugation: more precisely $C|n\rangle = |\bar{n}\rangle$
  - *C* changes sign of ‘internal’ quantum numbers
    - o charge, baryon number, lepton number, strangeness, charm,
    - o BUT: mass, energy, momentum, spin NOT affected
  - $C^2 = 1 \Rightarrow$ eigenvalues of *C* are +1, -1
- Note: most particles are NOT eigenstates of *C*
- only: particles which are their own antiparticle – photon, π^0, η, φ, … ψ
  C is multiplicative, conserved in electromagnetic and strong processes

Charge Conjugation

- Charge Conjugation is conserved in electromagnetic and strong Inter.
- Examples
  - o $\pi^0 \rightarrow \gamma + \gamma$; *C* for *n* photons $C = (-1)^n \Rightarrow \pi^0 \rightarrow 3\gamma$ forbidden; not observed
  - o $p + \overline{p} \rightarrow \pi^+ + \pi^- + \pi^0 \Rightarrow$ energy distribution for charged pions is on average identical

- Mesons: quark-antiquark system
  - o one can show: system of (S = ½ particle) • (antiparticle) has
    - eigenstates with $C = (-1)^{l+S}$
    - pseudoscalars: $l = 0,\ S = 0,\ C = 1$
    - vectors: $l = 0,\ S = 1,\ C = -1$

CP: charge- parity

- Remember the culture shock: weak interactions are not *P* invariant
  - $\pi^+ \rightarrow \mu^+ + \nu_\mu$
    - antimuon emitted is always left-handed→ ν_μ is left-handed

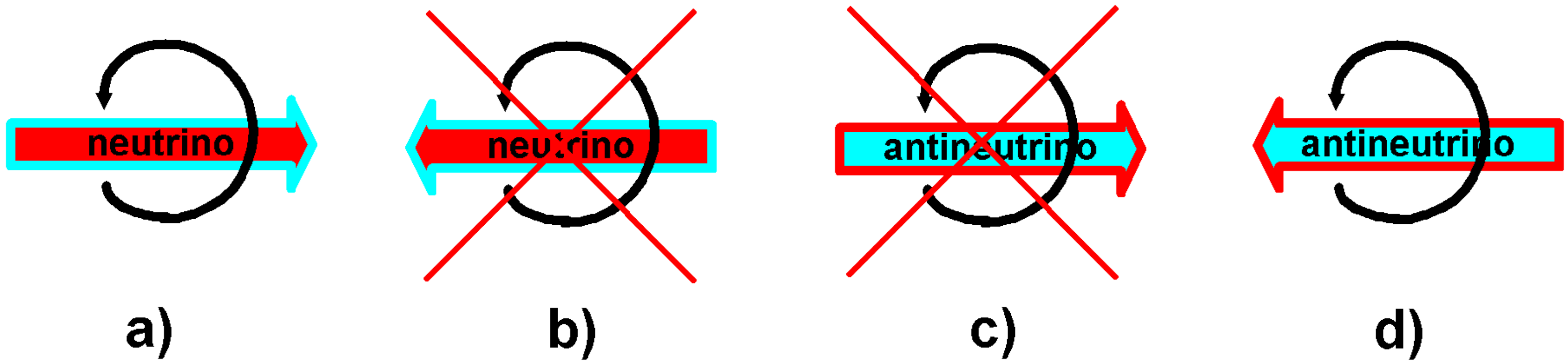
      (pion has s=0; muon and neutrino spins opposite)

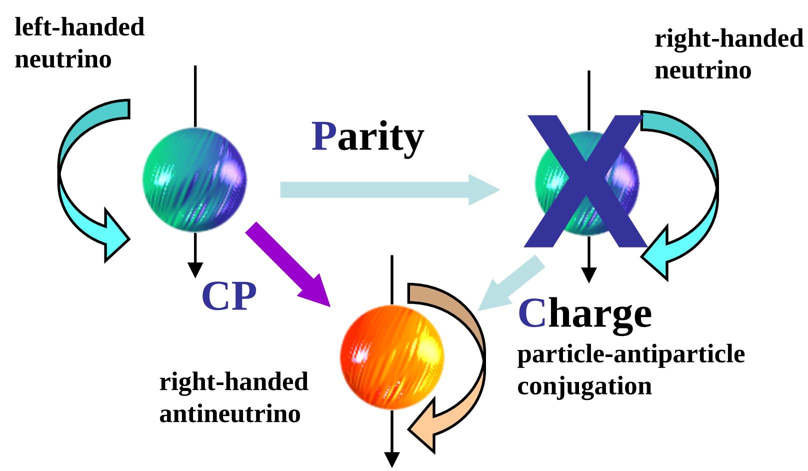
  Weak Interactions are also not invariant under C: charge-conjugated reaction
  - $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$ not possibe; μ^- is not left-handed; always right-handed

- BUT: under combined operation of CP
  - left-handed antimuon ⇒ right-handed muon
- Combined operation of CP seems to be the right symmetry operation
  - Pauli is happy – ‘die Welt ist wieder in Ordnung’ ……

    for a few years ……

spinning neutrinos and antineutrinos





In weak interactions P and C are "maximally violated" while the combined symmetry under CP is (almost) conserved.

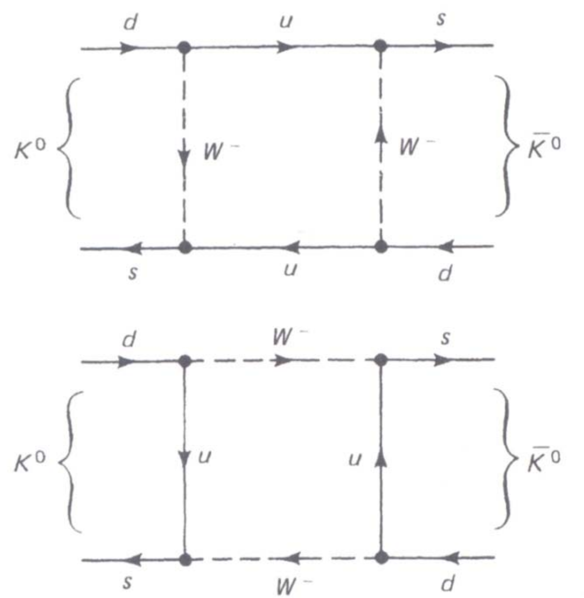
Puzzle, Mystery, Beauty of the neutral K-Meson System

- K^0 and $\overline{K}^0$ can be produced in strong interaction processes
  - $K^- + p \rightarrow \overline{K}^0 + n$; $K^+ n \rightarrow K^0 + p$;..
  - Kaons are produced in states of unique strangeness
  - $\overline{K}^0$ (S= -1) is antiparticle of K^0 (S=+1)
- Neutral kaons are unstable and decay through weak interaction
  - Experimentally observed: two different decay times !
- Only possible, if these states consist of a superposition of two distinct states with different lifetimes
  - a short-lived one, originally labeled K_1
  - a long-lived one, originally labeled K_2
- K^0 and $\overline{K}^0$ are eigenstates of the Strong Hamiltonian, but not eigenstates of the Weak Hamiltonian
- K_1 and K_2 are eigenstates of the weak Hamiltonian

Puzzle, Mystery, Beauty of the K-Meson System

- Gell-Mann and Pais (1955)
- noticed that K^0 (strangeness S= +1) can turn into antiparticle $K^0 \Leftrightarrow \overline{K}^0$, because both particle can decay into $\pi^+ + \pi^-$ through second order weak interaction

Feynman diagrams in modern formulation



- Particles, normally observed in laboratory, are linear combinations of these two states

Neutral K-System

- K's are pseudoscalars

$$P|K^0\rangle = -|K^0\rangle \qquad P|\overline{K}^0\rangle = -|\overline{K}^0\rangle$$
$$C|K^0\rangle = |\overline{K}^0\rangle \qquad C|\overline{K}^0\rangle = -|K^0\rangle$$
$$CP|K^0\rangle = -|\overline{K}^0\rangle \qquad CP|\overline{K}^0\rangle = -|K^0\rangle$$

- The normalized eigenstates of CP are

$$|K_1\rangle = \left(\tfrac{1}{\sqrt{2}}\right)\left(|K^0\rangle - |\overline{K}^0\rangle\right) \quad |K_2\rangle = \left(\tfrac{1}{\sqrt{2}}\right)\left(|K^0\rangle + |\overline{K}^0\rangle\right)$$
$$CP|K_1\rangle = |K_1\rangle \quad CP|K_2\rangle = -|K_2\rangle$$

- If CP is conserved in weak interactions
  - $K_1 \rightarrow$ can decay only in CP = +1 state
  - $K_2 \rightarrow$ can decay only in CP = -1 state
- Kaons typically decay into (P $|\pi^0\rangle$ = - $|\pi^0\rangle$ C $|\pi^0\rangle$ = $|\pi^0\rangle$)

2 π state (CP = +1)

3 π state (CP = -1)

- Conclusion: $K_1 \rightarrow 2\pi$, $K_2 \rightarrow 3\pi$

K_1, K_2

- 2π – decay is much faster (more energy released)
- Start with K^0-beam $|K^0\rangle = \left(\frac{1}{\sqrt{2}}\right)\left(|K_1\rangle - |K_2\rangle\right)$
  - $|K_1\rangle$ component will decay quickly, leaving more $|K_2\rangle$'s
- In Cronin's memoirs

So these gentlemen, Gell-Mann and Pais, predicted that in addition to the short-lived K mesons, there should be long-lived K mesons. They did it beautifully, elegantly and simply. I think theirs is a paper one should read sometimes just for its pure beauty of reasoning. It was published in the Physical Review in 1955. A very lovely thing! You get shivers up and down your spine, especially when you find you understand it. At the time, many of the most distinguished theoreticians thought this prediction was really ***baloney ('Unsinn').***

……. it was not baloney

- 1955: Lederman and collaborators discover K_2 meson

$$\tau_1 = 0.895 \times 10^{-10}\ \text{sec}$$

$$\tau_2 = 5.11 \times 10^{-8}\ \text{sec}$$

- Note: K_1 and K_2 are NOT antiparticles of one another
  (K_0 and $\overline{K}_0$ are antiparticles of one another)
  K_1 is its own antiparticle $C = -1$
  K_2 is its own antiparticle $C = +1$

- They differ by a tiny mass difference

$$m_2 - m_1 = 3.48 \times 10^{-6}\,\text{eV} \quad (\sim 10^{-11}\ \text{of electron mass})$$

What is a ‘Particle’ ?

- Kaons are produced by strong interactions, in eigenstates of strong Hamiltonian, in eigenstates of strangeness $\left(K^0 \, and \, \overline{K}^0\right)$

- Kaons decay by the weak interaction, as eigenstates of CP (K_1, K_2)

- What is the real particle ? Characterized by unique life time ?

- Analogy with polarized light
  - linear polarization can be regarded as superposition of left-circular and right-circular polarization
  - traversal of medium, which preferentially absorbs right-circularly polarized light ⇒ linearly polarized light will become left-polarized

$$\mathrm{K}^0 \text{ beams} \Rightarrow \mathrm{K}_2 \text{ beams}$$

The Cultural Revolution: CP violation

- 1964: Cronin, Fitch and collaborators observe CP violation
- K_0 - beam: by letting the K_1 component decay ⇒ can produce arbitrarily pure K_2 - beam; K_2 is a CP=-1 state; can only decay into CP=-1 (3 pions), if CP is conserved
- Observation:
  - observed: 22700 3π-decay; 45 2π-decay

- observed:	22700	3π-decay
	45	2π-decay

- Long-lived component is NOT perfect eigenstate of CP, contains a small admixture of K_1

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right) \quad |K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1\rangle + \varepsilon|K_2\rangle\right)$$

- measure of departure from perfect CP invariance is ε: $\varepsilon = 2.24 \times 10^{-3}$

Cronin et al experiment

- spark chambers
- scintillators
- Cerenkov detectors

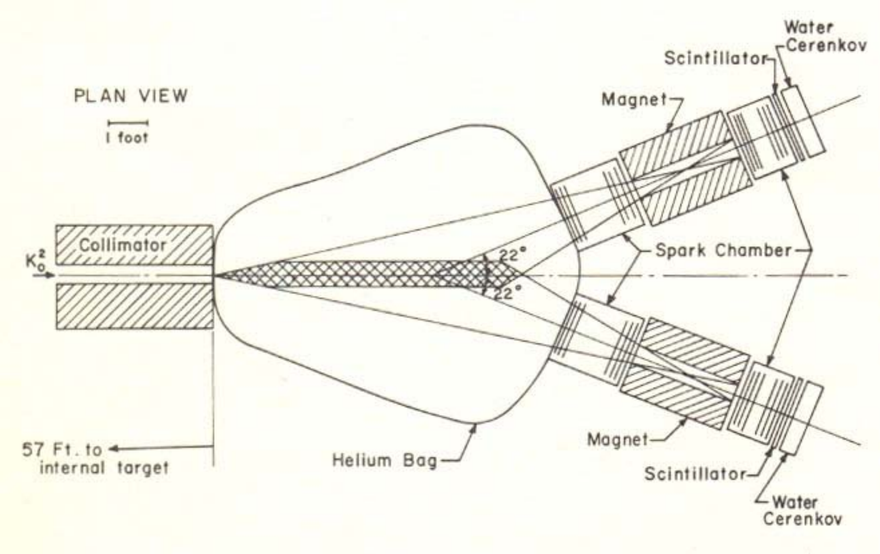


FIG. 1. Plan view of the detector arrangement.

the first signal:

$K_L \rightarrow \pi^+ \pi^-$

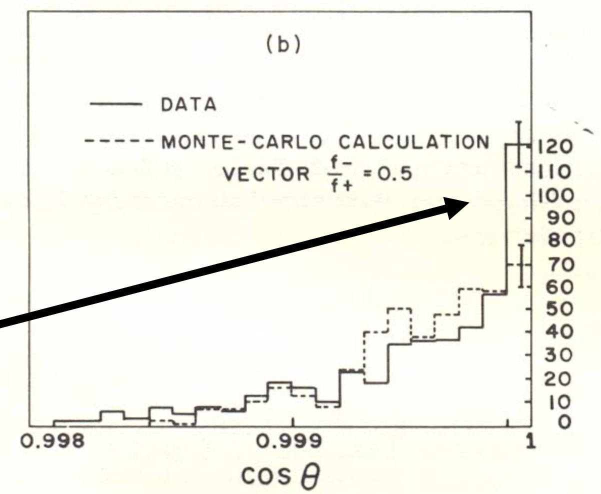


FIG. 2. (a) Experimental distribution in m^* compared with Monte Carlo calculation. The calculated distribution is normalized to the total number of observed events. (b) Angular distribution of those events in the range $490 < m^* < 510$ MeV. The calculated curve is normalized to the number of events in the complete sample.

CP-Violation

- Parity violation is treated in W.I. easily because it is maximally violated
- CP violation in contrast is a very small effect
- In the ‘Standard Model’ it was incorporated in the ‘Cabbibo-Kobayashi-Maskawa’ (CKM) mixing matrix ⇒
- 1973: Kobayashi, Maskawa: show how it could be incorporated, but requiring THREE generations of quarks ! (Nobel Prize in 2008)
- Even more dramatic
  - $K_L \rightarrow \pi^+ e^- + \bar{\nu}_e \ and \ (CP) \quad \pi^- + e^+ + \nu_e$
  - if CP is good symmetry: the two decay rates are equal
  - experimentally: rates differ by 1 in 3.3 x 10^{-3}
- Absolute distinction between Matter and Antimatter

‘Matter’: charge produced preferentially in the decay of K_L !

Beauty Factories

- CP violation occurs also in neutral *B*-meson system

- '*B*-factories': e^+e^- colliders, optimized for $B\bar{B}$ - production
  - constructed at SLAC (BaBar Experiment)
  - contstructed at KEK (BELLE Experiment)

- The precision experiments confirmed the CKM Theory ⇒ cited in the Nobel Prize Award

- HEPHY is a major partner in BELLE and
- Leading partner in BELLE II (aim for much higher sensitivity)
  - one research area with challenging opportunities for project diploma, dissertation work
    - talk to C. Schwanda (BELLE II Project Leader) or C. Fabjan

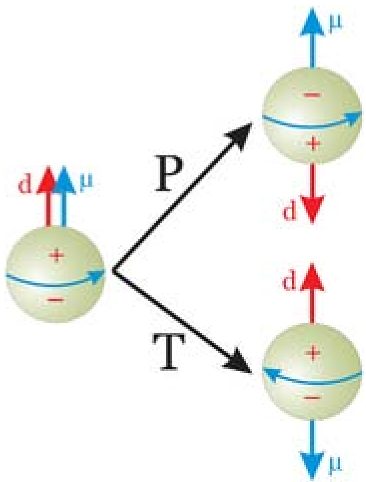
Time Reversal and CTP Theorem

- CP is violated: what about T invariance?

- T invariance very difficult to test experimentally
  - expected to be violated in W.I. ⇒ usually signal overwhelmed by em and strong interaction

- Classic example:
  - electric dipole moment of elementary particle (neutron)
  - *d* points along spin S
  - *d* is vector, S is pseudovector
  - $d \neq 0$ ⇒ violation of P
  - *S* changes sign under T, *d* does not
  - $d \neq 0$ ⇒ violation of T

Neutron Electric Dipole Moment (nEDM)

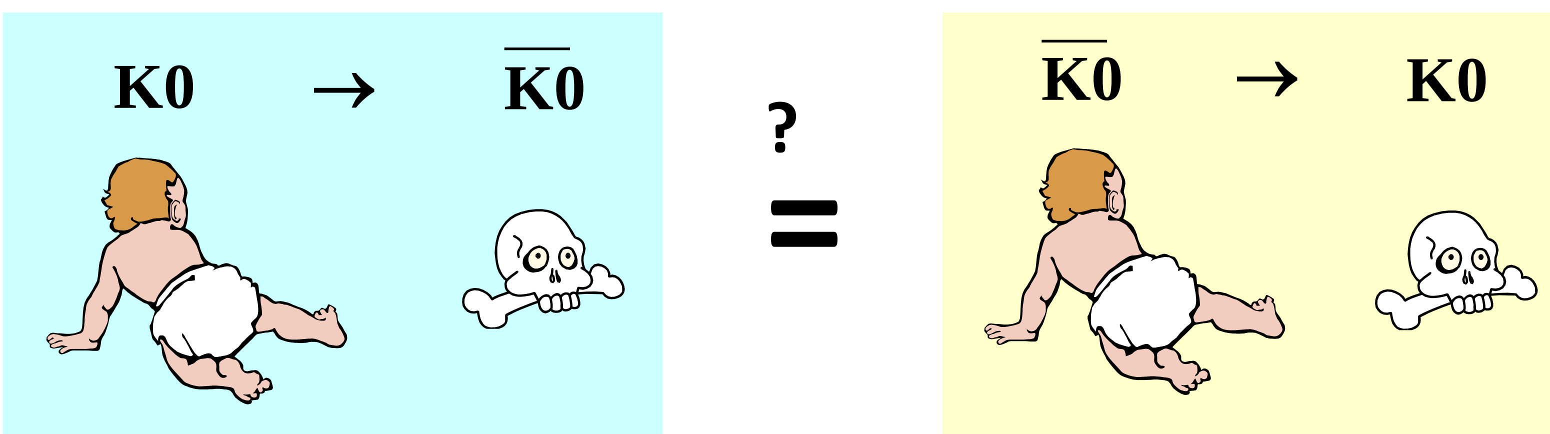


Assume neutron is globally neutral, but has positive and negative charge distribution resulting in electric dipole moment
Time reversal changes spin direction, but does not change charge distribution
→ nEDM does not change
nEDM has to be parallel to spin →
Conditions only satisfied, if nEDM=0
nEDM ≠ 0 → Time invariance is violated

Present limit nEDM< 3.10^{-26} e.cm
Standard Model (due to CP violation)
nEDM ≈ 10^{-32} e.cm

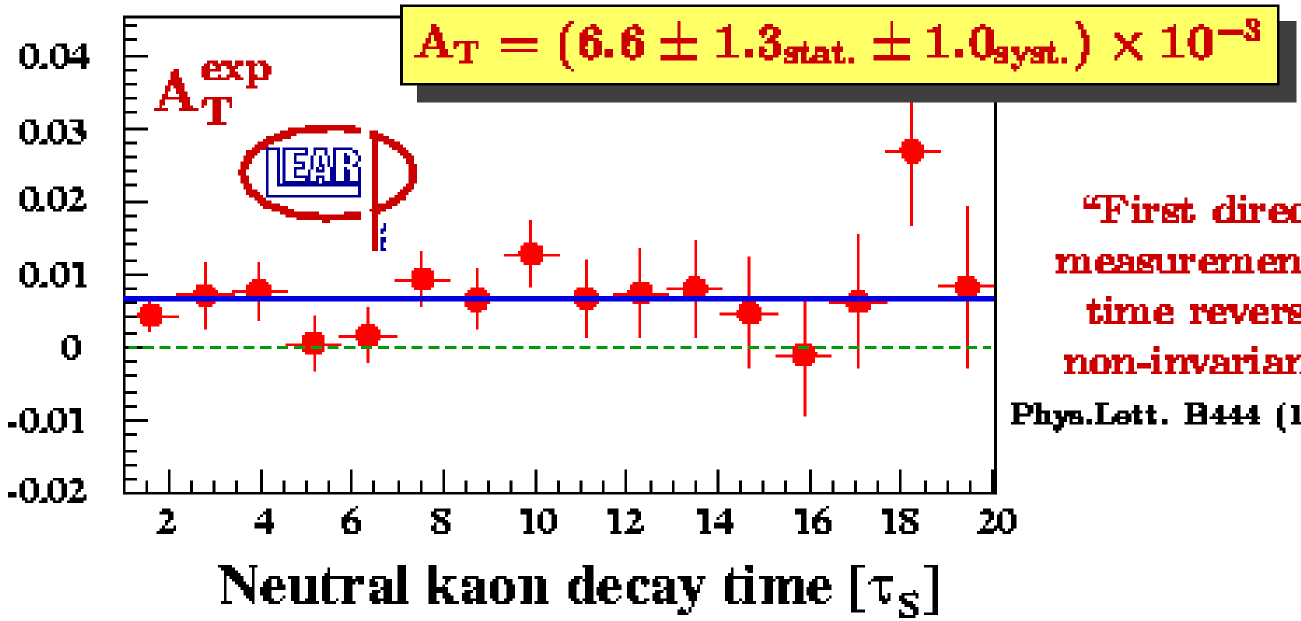
Measure T-violation by comparing $K^0 \rightarrow \overline{K}^0$ and $\overline{K}^0 \rightarrow K^0$

Compare rates for neutral kaons which are created as K^0 and decay as $\overline{K}^0$ with the inverse process:



Direct measurement of T-violation by CPLEAR at CERN

$$A_T = \frac{R\left(\overline{K^0} \to K^0\right) - R\left(K^0 \to \overline{K^0}\right)}{R\left(\overline{K^0} \to K^0\right) + R\left(K^0 \to \overline{K^0}\right)} = 4\Re e\, \varepsilon_T$$



"First direct" measurement of time reversal non-invariance!

Phys.Lett. B444 (1998) 43