Decays and Scattering

- Decay Rates
- Cross Sections
- Calculating Decays
- Scattering
- Lifetime of Particles

Decay Rates

- There are THREE experimental probes of Elementary Particle Interactions
  - bound states
  - decay of particles
  - scattering of particles

- Decay of a particle
  - is of course a statistical process
  - represented by the average (mean) life time

- Remember:
  - elementary particles have no memory
  - probability of a given muon decaying in the next microsecond is independent of time of creation of muon (humans are VERY different !)

- Decay Rate Γ: probability per unit time that particle will disintegrate

Decay Rates

- Large collection of ; for example, muons N(t)
- Decay rate $dN = -\Gamma N dt$

  $$N(t) = N(0)\exp(-\Gamma t)$$

- Mean lifetime $\tau = 1/\Gamma$
- Most particles will decay by several routes
  - total decay rate $\Gamma_{tot} = \Sigma\,\Gamma_i$; $\tau = 1/\Gamma_{tot}$
- Branching ratio for ith decay mode
  - $BR(i) = \Gamma_i / \Gamma_{tot}$

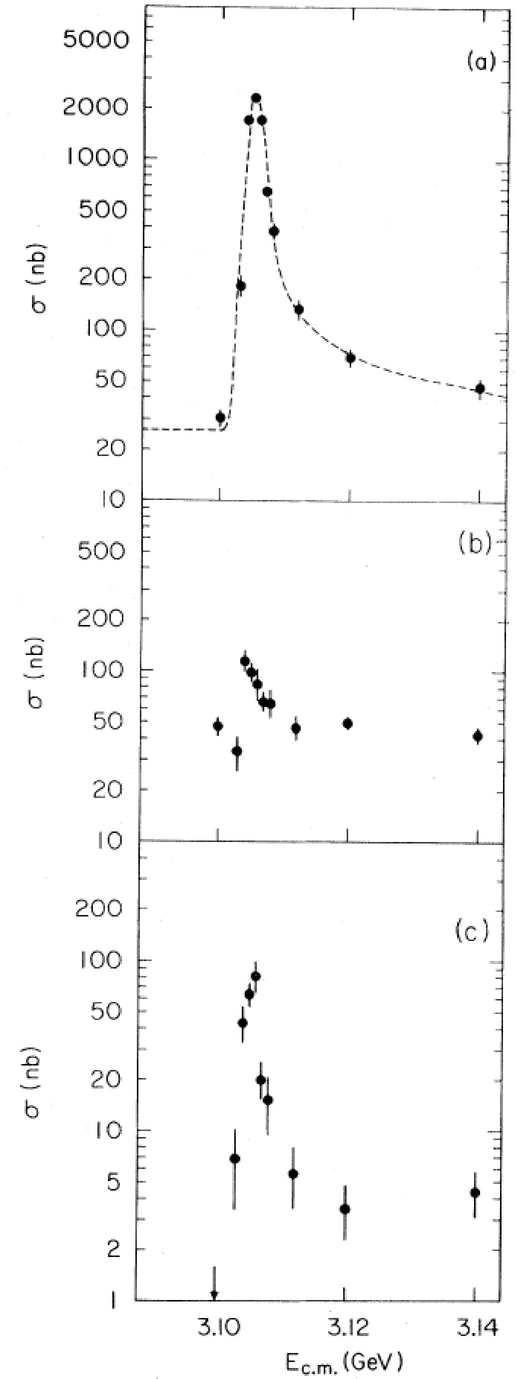
Decay Width Γ

- Example: decay width of the J/ψ
- Shown is the original measurement at the SPEAR e^+e^- Collider; the line width is larger (~ 3.4. MeV) due to energy spread in the beams
- Natural line width is 93 keV

- Lifetime is $\Delta E \, \Delta t \leq h/2\pi = 6.6 \; 10^{-16}$ eV

- $\Delta t \sim 7 \; 10^{-21}$ s

- We will see later – within a discussion of the quark model – why this lifetime is relatively long



Scattering: Cross sections

- What do we measure ? What do we calculate ?

- Classical analog:
  - aiming at target
  - what counts is the 'size' of the target

- Elementary particle scattering
  - size ('cross-sectional area') of particle is relevant
  - however: size is not 'rigidly' defined; size, area is 'fuzzy'
  - effect of 'collision' depends on distance between projectile and scattering center
  - we can define an 'effective cross section'

- Cross sections depend on
  - Nature and energy of projectile (electrons scatter off a proton more sharply than a neutrino)
  - nature of scattering result (elastic, inelastic $\Rightarrow$ many different possibilities)

Cross Sections

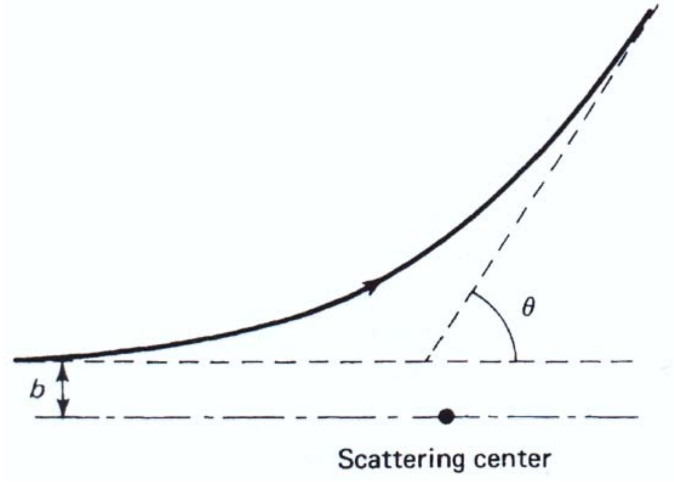
- Inelastic scattering: several final states
  - $e + p \rightarrow e + p + \gamma$; $\rightarrow e + p + \pi^0$; ….
  - each channel has its 'exclusive' scattering cross section
  - total ('inclusive') cross section $\sigma_{tot} = \Sigma\, \sigma_i$;

- Cross section depends in general on velocity of projectile
  - naively: σ proportional to time in vicinity of target $\Rightarrow \sigma \sim 1/v$
  - strong modification, if incident particle has energy to form a 'resonance' (or an excitation): a short-lived, quasi-bound state $\Rightarrow$ remember the discovery of the J/ψ in e^+e^- collisions at SLAC
  - Frequently used procedure to discover short-lived particles

Cross Sections

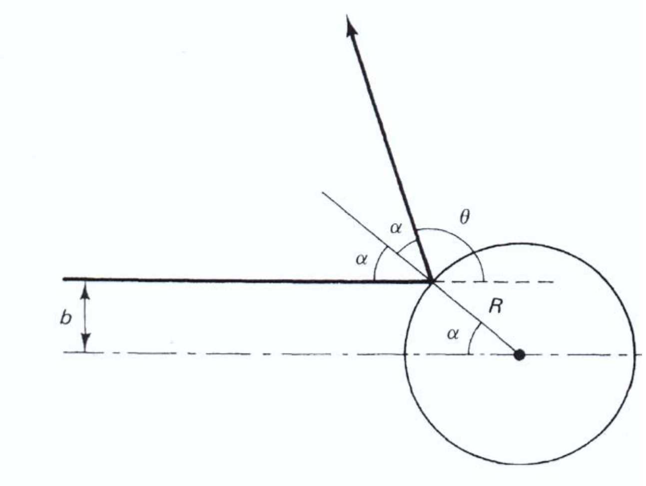
- Suppose particle (e.g. electron) scatters of a potential (e.g. Coulomb potential of stationary proton)

- Scattering angle θ is function of impact parameter b, i.e. the distance by which the incoming particle would have missed the scattering center



Hard-Sphere Scattering

- Particle scatters elastically on sphere with radius R

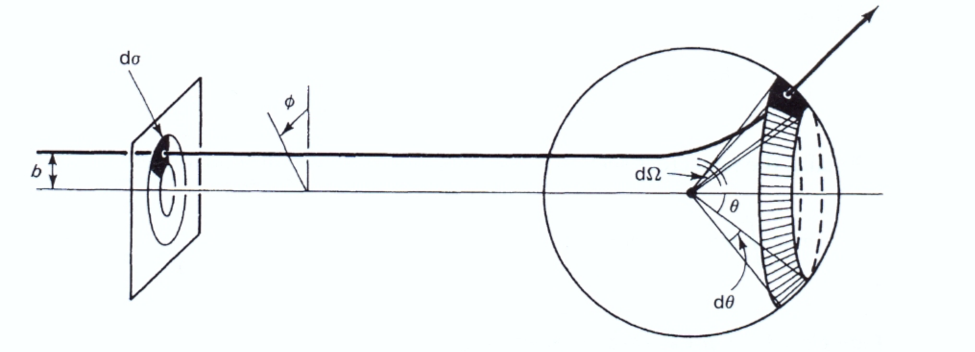


- $b = R \sin \alpha \quad 2\alpha + \theta = \pi$
  $\sin \alpha = \sin(\pi/2 - \theta/2) = \cos(\theta/2)$
  $b = R \cos(\theta/2) \quad \theta = 2 \cos^{-1}(b/R)$

- If particle impinges with impact parameter between b and b + db ⇒ will emerge under angle θ and $\theta + d\theta$

Scattering into Solid Angle dΩ

- More generally, if particle passes through an infinitesimal area dσ at impact parameter b, b+db ⇒ will scatter into corresponding solid angle dΩ



- Differential cross section (differential with respect a certain parameter: solid angle; momentum of secondary particle…)
- $$d\sigma = D(\theta)\, d\Omega$$
$$d\sigma = \left| b\, db\, d\phi \right|, \quad d\Omega = \left| \sin\theta\, d\theta d\phi \right|$$
$$D(\theta) = \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \left(\frac{db}{d\theta} \right) \right|$$

(areas, solid angles are intrinsically positive, hence absolute value signs)

Scattering: Some examples

- Hard-sphere scattering

$$\frac{db}{d\theta} = -\frac{R}{2}\sin\left(\frac{\theta}{2}\right)$$

$$D(\theta) = \frac{Rb\sin(\theta/2)}{2\sin\theta} = \frac{R^2}{2}\frac{\cos(\theta/2)\sin(\theta/2)}{\sin\theta} = \frac{R^2}{4}$$

- Total cross section $\sigma = \int d\sigma = \int D(\theta)\, d\Omega$
  – σ_{total} (hard-sphere) = $\int \frac{R^2}{4} d\Omega = \pi R^2$
  - for hard-sphere: πR^2 obviously presents the scattering area
- Concept of ‘ cross section’ applies also to ‘soft’ targets

Rutherford Scattering

- Particle with charge q_1 and energy E scatter off a stationary particle with charge q_2

- Classically: $b = \frac{q_1 q_2}{2E} \cot(\theta / 2)$

- Differential cross section: $D(\theta) = \left(\frac{q_1 q_2}{4E \sin^2(\theta/2)} \right)^2$

- Total cross section: $\sigma = 2\pi \left(\frac{q_1 q_2}{4E} \right)^2 \int_0^{\pi} \frac{1}{\sin^2(\theta/2)} \sin\theta \, d\theta = \infty$

  infinite, because Coulomb potential has infinite range

- Remember: in calculation of energy loss of a charged particle one imposes a cut off (b_{max}), beyond which not atomic excitations are energetically possible.

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Cowan, Reines Experiment of Neutrino Discovery: estimate of event rate

neutrino flux: 5* 10^{13} neutrinos/ cm^2 s

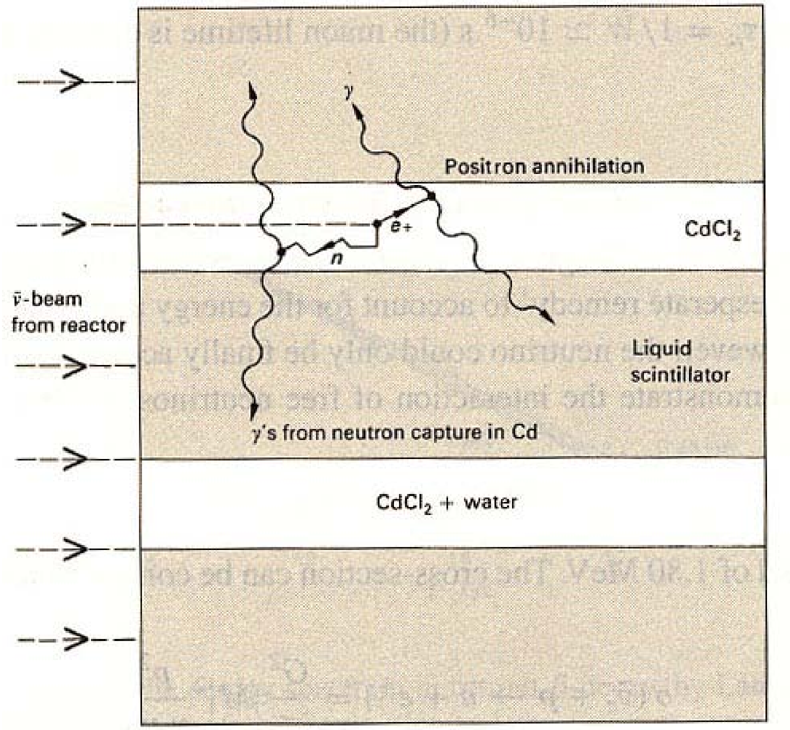
Tanks was located 11 m from reactor core, 12 m blow surface (why?)

v absorbed in water (200 l); effective area was 3000 cm2; depth ~ 70 cm

Cross section for absorption (inverse β-decay $\sigma = 6 * 10^{-44}\ cm^2$

Energy threshold for reaction: 1.8 MeV

3% of the reactor neutrinos are above threshold



Cowan, Reines Experiment of Neutrino Discovery: estimate of event rate

$N_A = 6.02 * 10^{23}$ / mol....18 g H_2O

Number of scatterers N_S/ cm^2

$N_S = N_A L \rho / 18 = 2.3 * 10^{24}$

Probability for interaction =

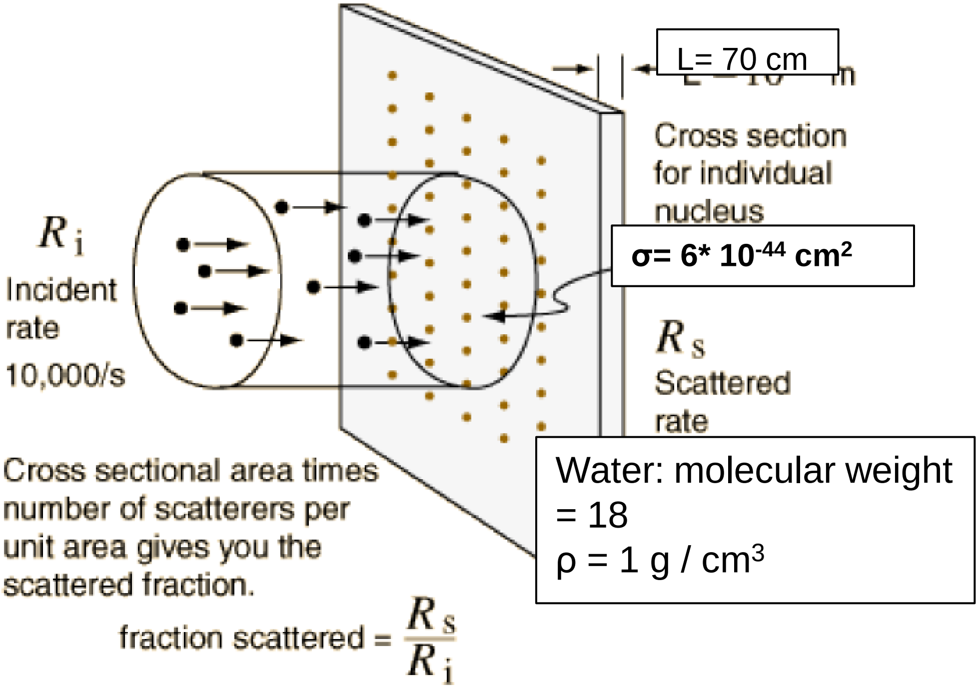
Effective 'cross section'/ cm^2 =

$P = N_S \sigma = 2.3 * 10^{24} * 6 * 10^{-44}$

$= 1.4 * 10^{-19}$

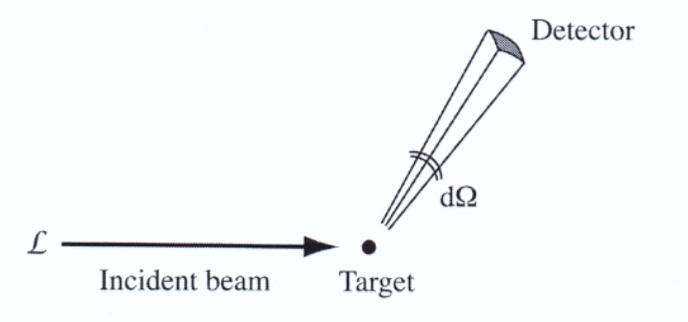
Rate= P * Flux *Detector Area * Threshold = 6.3 * 10^{-4}

Rate is ~ 2 events/ hour…was actually observed!



Luminosity

- Luminosity $\mathcal{L}$: number of particles per unit time and unit area: $[\,\mathcal{L}\,] = cm^{-2}\,s^{-1}$



- $dN = \mathcal{L}\, d\sigma$ …. number of particles per unit time passing through area $d\sigma$ and b, b+db, which is also number per unit time scattered into solid angle $d\Omega$: $dN = \mathcal{L}\, d\sigma = \mathcal{L}\, D(\theta)\, d\Omega$

- Setting up a scattering experiment with a detector covering a solid angle $d\Omega$ and counting the number of particles per unit time dN
- Measurement of differential cross section $\frac{d\sigma}{d\Omega} = \frac{dN}{\mathcal{L}\, d\Omega}$
- After integration over $d\Omega$ $\rightarrow$ $N = \sigma\mathcal{L}$
- At LHC: $\mathcal{L} = 10^{34}\, cm^{-2}\, s^{-1}$, $\sigma = 100$ mb$= 10^{-25}\ cm^2 \rightarrow N = 10^9$ coll/s

Calculating Decay Rates and Cross Sections

- Calculation needs two ingredients
  - the amplitude $\mathcal{M}$ for the process (dynamics)
  - the phase space available (kinematics)

- $\mathcal{M}$ depends on the physics, type of interaction, reaction, …

- Phase space: 'room to maneuver'
  - heavy particle decaying into light particles involves a large phase space factor
  - neutron decaying into $p + e^- + \overline{\nu}_e$ has a very small phase space factor

- Fermi 'Golden Rule'
  - transition rate = (phase space) • |amplitude|^2
  - Non-relativistic version formulated with perturbation theory
  - can be derived with relativistic quantum field theory

Golden Rule for Decays

- Particle 1 (at rest) decaying into 2, 3, 4, …. n particles

  $1 \rightarrow 2 + 3 + 4 + \dots + n$

- Decay rate Γ

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathrm{M}|^2 (2\pi)^4 \delta^4 (p_1 - p_2 - p_3 \ldots - p_n) \times \prod_{j=2}^{n} 2\pi \delta\left(p_j^2 - m_j^2 c^2 \right) \theta\left(p_j^0 \right) \frac{d^4 p_j}{(2\pi^4)}$$

- m_j, p_j is mass, four-momentum of jth particle ; S is statistical factor to account for identical particles in final state; (if no identical particles in final state S = 1)

- $|\mathcal{M}|^2$ is amplitude squared; contains the dynamics

- The rest is 'Phase Space'

Phase Space

- Integration over all outgoing four-momenta subject to three kinematical constraints
  - each outgoing particle lies on its mass shell: $\mathrm{p}_\mathrm{j}^2 = \mathrm{m}_\mathrm{j}^2 \mathrm{c}^2$

    i.e. $E_j^2 - \vec{p}_j^2 c^2 = m_j^2 c^4$, enforcedby $\delta\left(p_j^2 - m_j^2 c^2\right)$
  - each outgoing energy is positive: $\mathrm{p}_\mathrm{j}^0 = \mathrm{E}_\mathrm{j}/\mathrm{c} > 0$; this is ensured by x the θ – function ($\theta(\mathrm{x}) = 0$ for x<0; $\theta(\mathrm{x}) = 1$ for x>0)
  - energy and momentum is conserved $p_1 = p_2 + p_3 \ldots + p_n$; this is ensured by delta function $\delta^4\left(p_1 - p_2 - \ldots - p_n\right)$
- Practical points:
  - $d^4 p = dp^0 d^3 \vec{p}$
  - $\delta\left(p^2 - m^2 c^2\right) = \delta\left[(p^0)^2 - p^2 - m^2 c^2\right] \Rightarrow p^0 \text{integrals doable, using } \delta \text{ function}$

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathrm{M}|^2 (2\pi)^4 \delta^4\left(p_1 - p_2 - p_3 \ldots - p_n\right) \times \prod_{j=2}^{n} \frac{1}{2\sqrt{\vec{p}_j^2 + m_j^2 c^2}} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

Two-Particle Decays

- For two particles in final state, previous expression reduces to

$$\Gamma = \frac{S}{32\pi^2 \hbar m_1} \int |\mathrm{M}|^2 \frac{\delta^4(p_1 - p_2 - p_3)}{\sqrt{\vec{p}_2^{\,2} + m_2^2 c^2}\sqrt{\vec{p}_3^{\,2} + m_3^2 c^2}} d^3\vec{p}_2 \times d^3\vec{p}_3$$

- After several pages of algebra and integration

$$\Gamma = \frac{S|\vec{p}|}{8\pi\hbar m_1^2 c}|\mathrm{M}|^2$$

- Surprisingly simple: phase space integration can be carried out without knowing functional form of $\mathcal{M}$ ⇒ two-body decays are kinematically determined: particles have to emerge back-to-back with opposite three-momenta
- no longer true for decays into more than two particles

Golden Rule for Scattering

- Particles 1 and 2 collide, producing 3,4, …. n particles
  $1 + 2 \rightarrow 3 + 4 + \ldots n$: after performing p_j^0 integrals

$$\sigma = \frac{S\hbar^2}{4\sqrt{(p_1 \bullet p_2)^2 - (m_1 m_2 c^2)^2}} \int |\mathrm{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 \ldots - p_n) \times \prod_{j=3}^{n} \frac{1}{2\sqrt{\vec{p}_j^{\,2} + m_j^2 c^2}} \frac{d^3 \vec{p}_j}{(2\pi)^3}$$

  same definitions, procedures as for decay

- Two-body scattering in CM frame $1+2 \rightarrow 3+4$; $\vec{p}_2 = -\vec{p}_1$

  $$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(\mathrm{E}_1 + E_2)^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$ with $|\vec{p}_i|, |\vec{p}_f|$ the magnitude of either incoming or outgoing particle

- Dimensions and units
  - decay rate $\Gamma = 1/\tau$ [sec^{-1}];
  - cross section: area [cm^2], 1 barn = 10^{-24} cm^2
  - $ds/d\Omega$ … barns/steradian
  - $\mathcal{M}$ … units depend on total number of particles involved $[\mathcal{M}] = (mc)^{4-n}$